

thm_2Eset_relation_2Eirreflexive_reln_to_rel_conv
(TMXn5MksnqRLChUvDd8HsUFQY4TAECWJ2PW)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P))))$

Definition 5 We define `c_2Ebool_2E_2EIN` to be $\lambda A_{27a} : \iota.(\lambda V0x \in A_{27a}.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_2E21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) P)))$

Definition 8 We define `c_2Ebool_2E_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))))$

Definition 9 We define `c_2Epred_set_2EREL_RESTRICT` to be $\lambda A_{27a} : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1s \in 2.V1s$

Definition 10 We define `c_2Ebool_2E_2E_2F` to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define `c_2Ebool_2E_2E_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E_2F))$

Definition 12 We define `c_2ERelation_2Eirreflexive` to be $\lambda A_{27a} : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(ap (c_2Ebool_2E_2E_7E A_{27a} R))$

Definition 13 We define `c_2Ebool_2E_2E_2E_5C_2E_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Epair_2Eprod : $\iota \Rightarrow \iota \Rightarrow \iota$` be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (2)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Definition 15 We define $c_2Eset_relation_2Eirreflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 16 We define $c_2Eset_relation_2Ereln_to_rel$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2E$

Definition 17 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 18 We define $c_2Eset_relation_2Erel_to_reln$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x))))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \vee (\exists V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (23)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0xy \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\forall V1R \in ((\\
& \quad \quad 2^{A_27b})^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad \quad A_27a\ A_27b))\ V0xy)\ (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a \\
& \quad \quad A_27b)\ V1R))) \Leftrightarrow (p\ (ap\ (ap\ V1R\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0xy)) \\
& \quad \quad (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0xy))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).(\forall V1x \in \\
& \quad \quad A_27a.(\forall V2y \in A_27b.((p\ (ap\ (ap\ (ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad \quad A_27a\ A_27b)\ V0r)\ V1x)\ V2y)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad \quad A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))\ V0r))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27b})^{A_27a}).((ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a\ A_27b) \\
& \quad \quad V0R)) = V0R))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).((ap\ (c_2Eset_relation_2Erel_to_reln \\
& \quad \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a\ A_27b) \\
& \quad \quad V0r)) = V0r))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r1 \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).(\forall V1r2 \in \\
& \quad \quad (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).(((ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad \quad A_27a\ A_27b)\ V0r1) = (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a \\
& \quad \quad A_27b)\ V1r2)) \Leftrightarrow (V0r1 = V1r2))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R1 \in ((2^{A_27b})^{A_27a}).(\forall V1R2 \in ((2^{A_27b})^{A_27a}). \\
& \quad \quad (((ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a\ A_27b)\ V0R1) = \\
& \quad \quad (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a\ A_27b)\ V1R2)) \Leftrightarrow \\
& \quad \quad (V0R1 = V1R2))))
\end{aligned} \tag{34}$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_{.27a}\ A_{.27a})}). \\ (\forall V1s \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Eset_relation_2Eirreflexive \\ A_{.27a}\ V0r)\ V1s)) \Leftrightarrow (p\ (ap\ (c_2Erelation_2Eirreflexive\ A_{.27a})\ (\\ ap\ (ap\ (c_2Epred_set_2EREL_RESTRICT\ A_{.27a})\ (ap\ (c_2Eset_relation_2Ereln_to_rel \\ A_{.27a}\ A_{.27a})\ V0r))\ V1s)))))) \end{aligned}$$