

thm_2Eset_relation_2Eirreflexive_reln_to_rel_conv_UNIV (TMKzwhUFuGx7WnNFxESGsCU7U4k4HgYXkJs)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.27a \Rightarrow \forall A.\lambda 27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epair_2EABS_prod A.27a A.27b) (V1f V0x))$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Eset_relation_2Ereln_to_rel$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27b)})$

Definition 10 We define $c_2Epred_set_2EREL_RESTRICT$ to be $\lambda A.\lambda 27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1s$

Definition 11 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E\ V0t) c_Ebool_2E))$

Definition 13 We define $c_ERelation_2Eirreflexive$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_Ebool_2E_21\ 2) (\lambda V0x \in V0R.V0x = V0x))$

Definition 14 We define $c_Eset_relation_2Eirreflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Assume the following.

$$True \tag{3}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & ((ap (ap (c_Epred_set_2EREL_RESTRICT\ A_27a)\ V0R) (c_Epred_set_2EUNIV\ A_27a)) = V0R)) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & (\forall V1s \in (2^{A_27a}).((p (ap (ap (c_Eset_relation_2Eirreflexive\ A_27a)\ V0r)\ V1s)) \Leftrightarrow (p (ap (c_ERelation_2Eirreflexive\ A_27a)\ (\\ & ap (ap (c_Epred_set_2EREL_RESTRICT\ A_27a)\ (ap (c_Eset_relation_2Ereln_to_rel\ A_27a\ A_27a)\ V0r))\ V1s)))))) \end{aligned} \tag{6}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & ((p (ap (ap (c_Eset_relation_2Eirreflexive\ A_27a)\ V0r) (c_Epred_set_2EUNIV\ A_27a))) \Leftrightarrow (p (ap (c_ERelation_2Eirreflexive\ A_27a)\ (ap (c_Eset_relation_2Ereln_to_rel\ A_27a\ A_27a)\ V0r)))))) \end{aligned}$$