

thm\_2Eset\_\_relation\_2Elinear\_\_order\_\_dom\_\_rg  
(TMXB-  
TYMR6n9DCN21iTQdEG3Yg8ZiVwCyazK)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_2Ebool\_2E\_IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 11** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EUNION\ A\_27a\ V0s\ V1t))$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a\ V0P)))$

**Definition 14** We define  $c\_2Eset\_relation\_2Eantisym$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

**Definition 15** We define  $c\_2Eset\_relation\_2Etransitive$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

**Definition 16** We define  $c\_2Eset\_relation\_2Erangle$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 17** We define  $c\_2Eset\_relation\_2Edomain$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 18** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a\ V0s\ V1t))$

**Definition 19** We define  $c\_2Eset\_relation\_2Elinear\_order$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ (p\ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ (2^{A\_27a}).(\forall V2x \in A\_27a.(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a\ V0s\ V1t)) \\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a\ V0s\ V1t)))) \Leftrightarrow ((p\ (ap \\ (ap\ (c\_2Ebool\_2EIN\ A\_27a\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a\ V2x)\ V1t))))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\ (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\ A\_27a\ V0s\ V1t))\ V2u)) \Leftrightarrow \\ ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a\ V0s)\ V2u)) \wedge (p\ (ap\ (ap \\ (c\_2Epred\_set\_2ESUBSET\ A\_27a\ V1t)\ V2u)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s1 \in (2^{A\_27a}). (\forall V1s2 \in \\ (2^{A\_27a}). ((V0s1 = V1s2) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET \\ A\_27a)\ V0s1)\ V1s2)) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a) \\ V1s2)\ V0s1)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). \\ ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Eset\_relation\_2Edomain \\ A\_27a\ A\_27b)\ V1r))) \Leftrightarrow (\exists V2y \in A\_27b. (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a \\ A\_27b)\ V0x)\ V2y))\ V1r)))))) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0lo \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). \\ (\forall V1X \in (2^{A\_27a}). ((p\ (ap\ (ap\ (c\_2Eset\_relation\_2Elinear\_order \\ A\_27a)\ V0lo)\ V1X)) \Rightarrow ((ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ (ap \\ (c\_2Eset\_relation\_2Edomain\ A\_27a\ A\_27a)\ V0lo))\ (ap\ (c\_2Eset\_relation\_2Erangle \\ A\_27a\ A\_27a)\ V0lo)) = V1X)))) \end{aligned}$$