

thm_2Eset__relation_2Elinear__order__restrict (TMVKtd1VNDRYtQjLZjGLY3SowmMojeQUQYx)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E1$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2E2$ to be $(ap (c_2Ebool_2E_2E1 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E2))$

Definition 7 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E1 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 9 We define $c_2Epair_2E_2EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 10 We define `c_2Epred__set_2EINTER` to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2E$

Definition 11 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge \lambda y. y \in A \wedge x < y) \text{ of type } \iota \Rightarrow \iota).$

Definition 12 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c.2Epair_2ESND \\ A.27a A.27b \in (A.27b^{(ty_2Epair_2Eprod A.27a A.27b)}) \end{aligned} \quad (4)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c.2Epair_2EFST \\ A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \end{aligned} \quad (5)$$

Definition 13 We define `c_2Epair_2EUNCURRY` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c^{A-27a}$

Definition 14 We define `c_2Eset__relation_2Errestrict` to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Definition 15 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define `c_2Eset__relation_2Eantisym` to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Definition 17 We define `c_2Eset__relation_2Etransitive` to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Definition 18 We define `c_2Eset__relation_2Erangle` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Definition 19 We define `c_2Eset__relation_2Edomain` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Definition 20 We define `c_2Epred__set_2ESUBSET` to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Definition 21 We define `c_2Eset__relation_2Elinear__order` to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)}$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0y \in A.27a. (\forall V1P \in \\
& (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V0y)\ (ap\ (c.2Epred_set_2EGSPEC \\
& A.27a\ A.27a)\ (\lambda V2x \in A.27a. (ap\ (ap\ (c.2Epair_2E.2C\ A.27a\ 2) \\
& V2x)\ (ap\ V1P\ V2x)))))) \Leftrightarrow (p\ (ap\ V1P\ V0y))))
\end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0x \in A.27a. (\forall V1y \in A.27b. (\forall V2P \in ((2^{A.27b})^{A.27a}). \\
& ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ (ty.2Epair_2Eprod\ A.27a\ A.27b))\ (ap \\
& (ap\ (c.2Epair_2E.2C\ A.27a\ A.27b)\ V0x)\ V1y))\ (ap\ (c.2Epred_set_2EGSPEC \\
& (ty.2Epair_2Eprod\ A.27a\ A.27b)\ (ty.2Epair_2Eprod\ A.27a\ A.27b)) \\
& (ap\ (c.2Epair_2EUNCURRY\ A.27a\ A.27b\ (ty.2Epair_2Eprod\ (ty.2Epair_2Eprod \\
& A.27a\ A.27b)\ 2))\ (\lambda V3x \in A.27a. (\lambda V4y \in A.27b. (ap\ (ap\ (c.2Epair_2E.2C \\
& (ty.2Epair_2Eprod\ A.27a\ A.27b)\ 2)\ (ap\ (ap\ (c.2Epair_2E.2C\ A.27a \\
& A.27b)\ V3x)\ V4y))\ (ap\ (ap\ V2P\ V3x)\ V4y)))))) \Leftrightarrow (p\ (ap\ (ap\ V2P\ V0x) \\
& V1y))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\
& V2x)\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap \\
& (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool_2EIN \\
& A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{11}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1r \in \\
& (2^{(ty.2Epair_2Eprod\ A.27a\ A.27a)}). (\forall V2s.27 \in (2^{A.27a}). \\
& ((p\ (ap\ (ap\ (c.2Eset_relation_2Elinear_order\ A.27a)\ V1r)\ V0s)) \Rightarrow \\
& (p\ (ap\ (ap\ (c.2Eset_relation_2Elinear_order\ A.27a)\ (ap\ (ap\ (\\
& c.2Eset_relation_2Errestrict\ A.27a)\ V1r)\ V2s.27))\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& A.27a)\ V0s)\ V2s.27))))))
\end{aligned}$$