

# thm\_2Eset\_relation\_2Emaximal\_union (TMEx58y2VaC4tLmoL8irSfpmfw2wZeLjfb7)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define  $c\_2Ebool\_2E\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(3)

**Definition 13** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Eset\_relation\_2Emaximal\_elements$  to be  $\lambda A\_27a : \iota.\lambda V0xs \in (2^{A\_27a}).\lambda V1r \in$

Assume the following.

$$True$$
(4)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))$$
(5)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t))))$$
(6)

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))$$
(7)

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))$$
(8)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$
(9)

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t))))))$$
(10)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))$$
(11)

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \quad (12)$$

$$(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow (\quad (13)$$

$$\forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in$$

$$A\_27b. (((ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V0x) \ V1y) = (ap \ (ap$$

$$(c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow (\quad (14)$$

$$\forall V0f \in ((ty\_2Epair\_2Eprod \ A\_27a \ 2)^{A\_27b}). (\forall V1v \in$$

$$A\_27a. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V1v) \ (ap \ (c\_2Epred\_set\_2EGSPEC$$

$$A\_27a \ A\_27b) \ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap \ (ap \ (c\_2Epair\_2E\_2C$$

$$A\_27a \ 2) \ V1v) \ c\_2Ebool\_2ET) = (ap \ V0f \ V2x))))))$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \quad (15)$$

$$(2^{A\_27a}). (\forall V2x \in A\_27a. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a)$$

$$V2x) \ (ap \ (ap \ (c\_2Epred\_set\_2EUNION \ A\_27a) \ V0s) \ V1t))) \Leftrightarrow ((p \ (ap$$

$$(ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V2x) \ V0s)) \vee (p \ (ap \ (ap \ (c\_2Ebool\_2EIN$$

$$A\_27a) \ V2x) \ V1t))))))$$

### Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0e \in A\_27a. (\forall V1s \in \quad (16)$$

$$(2^{A\_27a}). (\forall V2r1 \in (2^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27a)}).$$

$$(\forall V3r2 \in (2^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27a)}). ((p \ (ap \ (ap$$

$$(c\_2Ebool\_2EIN \ A\_27a) \ V0e) \ (ap \ (ap \ (c\_2Eset\_relation\_2Emaximal\_elements$$

$$A\_27a) \ V1s) \ (ap \ (ap \ (c\_2Epred\_set\_2EUNION \ (ty\_2Epair\_2Eprod$$

$$A\_27a \ A\_27a) \ V2r1) \ V3r2)))) \Rightarrow ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a)$$

$$V0e) \ (ap \ (ap \ (c\_2Eset\_relation\_2Emaximal\_elements \ A\_27a) \ V1s)$$

$$V2r1))) \wedge (p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V0e) \ (ap \ (ap \ (c\_2Eset\_relation\_2Emaximal\_elements$$

$$A\_27a) \ V1s) \ V3r2))))))$$