

thm_2Eset_relation_2Eminimal_elements_mono (TMduiVGVH2Bv58nebeyVTJsUvjGQvn6iGTg)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define c_2Ebool_2EET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2)$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 9 We define $c_2Eset_relation_2Eminimal_elements$ to be $\lambda A_27a : \iota.\lambda V0xs \in (2^{A_27a}).\lambda V1r \in (2^{2^{A_27a}})$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0y \in A_{27a}. (\forall V1P \in \\
& (2^{A_{27a}}). ((p (ap (ap (c_2Ebool_2EIN A_{27a}) V0y) (ap (c_2Epred_set_2EGSPEC \\
& A_{27a} A_{27a}) (\lambda V2x \in A_{27a}. (ap (ap (c_2Epair_2E_2C A_{27a} 2) \\
& V2x) (ap V1P V2x)))))) \Leftrightarrow (p (ap V1P V0y))))))
\end{aligned} \tag{4}$$

Theorem 1

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod A_{27a} A_{27a})}). \\
& (\forall V1r_{27} \in (2^{(ty_2Epair_2Eprod A_{27a} A_{27a})}). (\forall V2xs \in \\
& (2^{A_{27a}}). ((p (ap (ap (c_2Epred_set_2ESUBSET (ty_2Epair_2Eprod \\
& A_{27a} A_{27a})) V0r) V1r_{27})) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET \\
& A_{27a} A_{27a}) (ap (ap (c_2Eset_relation_2Eminimal_elements A_{27a}) \\
& V2xs) V1r_{27})) (ap (ap (c_2Eset_relation_2Eminimal_elements \\
& A_{27a} V2xs) V0r))))))
\end{aligned}$$