

# thm\_2Eset\_relation\_2Epartial\_order\_reln\_to\_rel\_conv\_UNIV (TMZ8ingxaSR7EuAWQjQCwtFLYRGqHZ4R815)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2EF$

**Definition 7** We define  $c\_2Ebool\_2E\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2ET)$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define  $c\_2Ebool\_2E\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t))$

**Definition 11** We define  $c\_2Erelation\_2ERUNION$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27b})^{A\_27a}).\lambda V1R2 \in ((2^{A\_27a})^{A\_27b}).$

**Definition 12** We define  $c\_2Eset\_relation\_2ERREFL\_EXP$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s \in (2^{A\_27a}).$

**Definition 13** We define  $c\_2Erelation\_2Etransitive$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_2E2F\_5C$

**Definition 14** We define  $c\_2Erelation\_2Eantisymmetric$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_2E2F\_5C$

**Definition 15** We define  $c\_2Erelation\_2Ereflexive$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_2E2F\_5C$

**Definition 16** We define  $c\_2Erelation\_2EWeakOrder$  to be  $\lambda A\_27g : \iota.\lambda V0Z \in ((2^{A\_27g})^{A\_27g}).(ap (ap c\_2Ebool\_2E\_2E2F\_5C$

**Definition 17** We define  $c\_2Eset\_relation\_2ERRUNIV$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(\lambda V1x \in A\_27a.(\lambda$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

**Definition 19** We define  $c\_2Eset\_relation\_2Ereln\_to\_rel$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 20** We define  $c\_2Erelation\_2ERSUBSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A-27b})^{A-27a}).\lambda V$

**Definition 21** We define  $c\_2Eset\_relation\_2Eantisym$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 22** We define  $c\_2Eset\_relation\_2Ereflexive$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 23** We define  $c\_2Eset\_relation\_2Etransitive$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 24** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 25** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a\ P))))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A-27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{2^{A-27b}})}) \quad (3)$$

**Definition 26** We define  $c\_2Eset\_relation\_2Erange$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 27** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2Emin\_2E\_40\ A\_27a\ s)\ t))$

**Definition 28** We define  $c\_2Eset\_relation\_2Edomain$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 29** We define  $c\_2Eset\_relation\_2Epartial\_order$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (5)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (6)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (7)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (9)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (10)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (11)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (12)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p \ (ap \ (ap \ (c\_2Ebool\_2EIN \\
A\_27a) \ V0x) \ (c\_2Epred\_set\_2EUNIV \ A\_27a)))) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((ap \ (ap \ (c\_2Eset\_relation\_2ERREFL\_EXP \ A\_27a) \ V0R) \ (c\_2Epred\_set\_2EUNIV \\
& A\_27a)) = V0R)) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0r \in (2^{(\text{ty\_2Epair\_2Eprod } A_{27a} \ A_{27a})}). \\
& \quad (\forall V1s \in (2^{A_{27a}}). ((p (ap (ap (c\_2Eset\_relation\_2Epartial\_order \\
& \quad A_{27a}) \ V0r) \ V1s)) \Leftrightarrow ((p (ap (ap (c\_2Erelation\_2ERSUBSET \ A_{27a} \ A_{27a}) \\
& \quad (ap (c\_2Eset\_relation\_2Ereln\_to\_rel \ A_{27a} \ A_{27a}) \ V0r)) (ap \\
& \quad (c\_2Eset\_relation\_2ERRUNIV \ A_{27a}) \ V1s))) \wedge (p (ap (c\_2Erelation\_2EWeakOrder \\
& \quad A_{27a}) (ap (ap (c\_2Eset\_relation\_2ERREFL\_EXP \ A_{27a}) (ap (c\_2Eset\_relation\_2Ereln\_to\_rel \\
& \quad A_{27a} \ A_{27a}) \ V0r)) \ V1s))))))
\end{aligned} \tag{15}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0r \in (2^{(\text{ty\_2Epair\_2Eprod } A_{27a} \ A_{27a})}). \\
& \quad ((p (ap (ap (c\_2Eset\_relation\_2Epartial\_order \ A_{27a}) \ V0r) ( \\
& \quad c\_2Epred\_set\_2EUNIV \ A_{27a})) \Leftrightarrow (p (ap (c\_2Erelation\_2EWeakOrder \\
& \quad A_{27a}) (ap (c\_2Eset\_relation\_2Ereln\_to\_rel \ A_{27a} \ A_{27a}) \ V0r))))))
\end{aligned}$$