thm_2Eset__relation_2Ereln__rel__conv__thms (TMWXt5mY4FEMojkyhdtSLepu5hKMtLUB8ym)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{4}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P\in 2^{A}.$**if** $(\exists x\in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x\in A\wedge p$
of type $\iota\Rightarrow\iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 10 *We define* c_2Erelation_2ERDOM *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R\in((2^{A\_27b})^{A\_27a}).\lambda V1x$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})\tag{5}$$

Definition 11 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod}$

Definition 12 *We define* c_2Erelation_2ERRANGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R\in((2^{A\_27b})^{A\_27a}).\lambda V1$

Definition 13 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A}$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t\in 2.V0t))$.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Erelation_2ESTRORD *to be* $\lambda A\_27a:\iota.\lambda V0R\in((2^{A\_27a})^{A\_27a}).\lambda V1a\in A\_27a.$

Definition 17 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in((A\_27c^{A\_27}$

Definition 18 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 19 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27}$

Definition 20 *We define* c_2Erelation_2EO *to be* $\lambda A\_27g:\iota.\lambda A\_27h:\iota.\lambda A\_27k:\iota.\lambda V0R1\in((2^{A\_27k})^{A\_27h})$

Definition 21 *We define* c_2Eset__relation_2Ercomp *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0r1\in(2^{(ty\_2E}$

Definition 22 *We define* c_2Eset__relation_2ERRUNIV *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(\lambda V1x\in A\_27a.(\lambda$

Definition 23 *We define* c_2Eset__relation_2Euniv__reln *to be* $\lambda A\_27a:\iota.\lambda V0xs\in(2^{A\_27a}).(ap\ (c\_2Epred\_\_s$

Definition 24 *We define* c_2Eset__relation_2Erel__to__reln *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R\in((2^{A\_27b})^{A\_27a}$

Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Definition 26 *We define* c_2Eset__relation_2Etc *to be* $\lambda A\_27a:\iota.(\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).($

Definition 27 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a:\iota.\lambda V0R\in((2^{A\_27a})^{A\_27a}).\lambda V1a\in A\_27a.\lambda V2b$

Definition 28 *We define* c_2Eset__relation_2Eacyclic *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a}$

Definition 29 *We define* c_2Epred__set_2EREL__RESTRICT *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s$

Definition 30 *We define* c_2Erelation_2Eirreflexive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2

Definition 31 *We define* c_2Eset__relation_2Eirreflexive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A}$

Definition 32 *We define* c_2Erelation_2ERUNION *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27b})^{A\_27a}).\lambda V1$

Definition 33 *We define* c_2Eset__relation_2ERREFL__EXP *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1s$

Definition 34 *We define* c_2Erelation_2Ereflexive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E

Definition 35 *We define* c_2Eset__relation_2Ereflexive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Definition 36 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2

Definition 37 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A}$

Definition 38 *We define* c_2Erelation_2Eantisymmetric *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Eb

Definition 39 *We define* c_2Eset__relation_2Eantisym *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Definition 40 *We define* c_2Erelation_2EWeakOrder *to be* $\lambda A\_27g : \iota.\lambda V0Z \in ((2^{A\_27g})^{A\_27g}).(ap\ (ap$ c_2Eb

Definition 41 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap$ (

Definition 42 *We define* c_2Eset__relation_2Epartial__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27}$

Definition 43 *We define* c_2Erelation_2Etrichotomous *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebo

Definition 44 *We define* c_2Erelation_2EWeakLinearOrder *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (ap$

Definition 45 *We define* c_2Eset__relation_2Elinear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a}$

Definition 46 *We define* c_2Eset__relation_2Ereln__to__rel *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2E}$

Definition 47 *We define* c_2Erelation_2EStrongOrder *to be* $\lambda A\_27g : \iota.\lambda V0Z \in ((2^{A\_27g})^{A\_27g}).(ap\ (ap$ c_2E

Definition 48 *We define* c_2Erelation_2EStrongLinearOrder *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (a$

Definition 49 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 50 *We define* c_2Eset__relation_2Estrict__linear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eproc}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2ERREFL\_\_EXP}\ A\_27a)\ V0R)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)) = V0R)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}\ A\_27a)\ V0R)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)) = V0R)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0xy \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\forall V1R \in ((2^{A\_27b})^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0xy)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ V1R))) \Leftrightarrow (p\ (ap\ (ap\ V1R\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0xy))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0xy)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ V0r)\ V1x)\ V2y)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y))\ V0r)))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27b})^{A\_27a}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ V0R)) = V0R)) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ V0r)) = V0r)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(\forall V1r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(((ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ V0r1) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R1 \in ((2^{A\_27b})^{A\_27a}).(\forall V1R2 \in ((2^{A\_27b})^{A\_27a}). (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ V0R1) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ V1R2)) \Leftrightarrow (V0R1 = V1R2)))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27b)\ V0r) = (ap\ (\mathsf{c\_2Erelation\_2ERDOM}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ V0r)))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27b)\ V0r) = (ap\ (\mathsf{c\_2Erelation\_2ERRANGE}\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27b\ A\_27a)\ V0r)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ V0r) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ESTRORD}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))\ V1s))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)}). (\forall V1r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27c\ A\_27b)}).((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ercomp}\ A\_27a\ A\_27b\ A\_27c)\ V0r1)\ V1r2) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27c\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27c\ A\_27b)\ V1r2))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27c)\ V0r1)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Euniv\_\_reln}\ A\_27a)\ V0s) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2ERRUNIV}\ A\_27a)\ V0s)))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).((ap\ (\mathsf{c\_2Eset\_\_relation\_2Etc}\ A\_27a)\ V0r) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eacyclic}\ A\_27a)\ V0r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eirreflexive}\ A\_27a)\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r)))))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eirreflexive}\ A\_27a)\ V0r)\ V1s)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eirreflexive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))\ V1s)))))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereflexive}\ A\_27a)\ V0r)\ V1s)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Ereflexive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2ERREFL\_\_EXP}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))\ V1s)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Etransitive}\ A\_27a)\ V0r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eantisym}\ A\_27a)\ V0r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eantisymmetric}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Epartial\_\_order}\ A\_27a)\ V0r)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EWeakOrder}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0r)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EWeakLinearOrder}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict\_\_linear\_\_order}\ A\_27a)\ V0r)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EStrongLinearOrder}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V0r))))) \tag{28}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.$$
$$nonempty\ A\_27c \Rightarrow (\forall V0xy \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).$$
$$(\forall V1R \in ((2^{A\_27b})^{A\_27a}).(\forall V2r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).$$
$$(\forall V3x \in A\_27a.(\forall V4y \in A\_27b.(\forall V5r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).$$
$$(\forall V6r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).(\forall V7R1 \in$$
$$((2^{A\_27b})^{A\_27a}).(\forall V8R2 \in ((2^{A\_27b})^{A\_27a}).(\forall V9R \in$$
$$((2^{A\_27a})^{A\_27a}).(\forall V10r \in (2^{(ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)}).$$
$$(\forall V11r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V12s \in$$
$$(2^{A\_27a}).(\forall V13r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)}).$$
$$(\forall V14r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27c\ A\_27b)}).((((p\ (ap$$
$$(ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0xy)\ (ap\ ($$
$$\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27b)\ V1R))) \Leftrightarrow (p\ (ap$$
$$(ap\ V1R\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0xy))\ (ap\ (c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b)\ V0xy)))) \wedge (((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27b)\ V2r)\ V3x)\ V4y)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod$$
$$A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V3x)\ V4y))\ V2r))) \wedge$$
$$(((ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}$$
$$A\_27a\ A\_27b)\ V1R)) = V1R) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}$$
$$A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)$$
$$V2r)) = V2r) \wedge ((((ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a$$
$$A\_27b)\ V5r1) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27b)$$
$$V6r2)) \Leftrightarrow (V5r1 = V6r2)) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}$$
$$A\_27a\ A\_27b)\ V7R1) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a$$
$$A\_27b)\ V8R2)) \Leftrightarrow (V7R1 = V8R2))))))) \wedge (((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2ERREFL\_\_EXP}$$
$$A\_27a)\ V9R)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)) = V9R) \wedge (((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}$$
$$A\_27a)\ V9R)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)) = V9R) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}$$
$$A\_27a\ A\_27b)\ V2r) = (ap\ (\mathsf{c\_2Erelation\_2ERDOM}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27b)\ V2r))) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27b)$$
$$V10r) = (ap\ (\mathsf{c\_2Erelation\_2ERRANGE}\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27b\ A\_27a)\ V10r))) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)$$
$$V11r) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)\ ($$
$$ap\ (\mathsf{c\_2Erelation\_2ESTRORD}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r)))) \wedge (((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}$$
$$A\_27a)\ V11r)\ V12s) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a$$
$$A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r))\ V12s))) \wedge (((ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ercomp}$$
$$A\_27a\ A\_27b\ A\_27c)\ V13r1)\ V14r2) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}$$
$$A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2EO}\ A\_27a\ A\_27c\ A\_27b)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27c\ A\_27b)\ V14r2))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27c)\ V13r1)))) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Euniv\_\_reln}$$
$$A\_27a)\ V12s) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)$$
$$(ap\ (\mathsf{c\_2Eset\_\_relation\_2ERRUNIV}\ A\_27a)\ V12s))) \wedge (((ap\ (\mathsf{c\_2Eset\_\_relation\_2Etc}$$
$$A\_27a)\ V11r) = (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erel\_\_to\_\_reln}\ A\_27a\ A\_27a)$$
$$(ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r)))) \wedge (((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eacyclic}$$
$$A\_27a)\ V11r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eirreflexive}\ A\_27a)\ (ap\ ($$
$$\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r))))) \wedge (((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eirreflexive}$$
$$A\_27a)\ V11r)\ V12s)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eirreflexive}\ A\_27a)$$
$$(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EREL\_\_RESTRICT}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r))\ V12s)))) \wedge (((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereflexive}$$
$$A\_27a)\ V11r)\ V12s)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Ereflexive}\ A\_27a)\ ($$
$$ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2ERREFL\_\_EXP}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r))\ V12s)))) \wedge (((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Etransitive}$$
$$A\_27a)\ V11r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}$$
$$A\_27a\ A\_27a)\ V11r)))) \wedge (((p\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Eantisym}$$
$$A\_27a)\ V11r)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Eantisymmetric}\ A\_27a)\ (ap$$
$$(\mathsf{c\_2Eset\_\_relation\_2Ereln\_\_to\_\_rel}\ A\_27a\ A\_27a)\ V11r)))) \wedge ($$
$$((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Epartial\_\_order}\ A\_27a)\ V11r)$$