thm_2Eset__relation_2Estrict__linear__order__restrict (TM-WoaT4nRGauD6J3F7inrb3NdydG65D2RXn)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 8 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 9 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c\_2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P\in 2^{A}.$**if** $(\exists x\in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x\in A\wedge p$
of type $\iota\Rightarrow\iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $c\_2Epair\_2ESND:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b\in(A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Let $c\_2Epair\_2EFST:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b\in(A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 12 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in((A\_27c^{A\_27}$

Definition 13 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t\in$

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 16 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_}$

Definition 17 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A}$

Definition 18 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod}$

Definition 19 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ ($

Definition 20 *We define* c_2Eset__relation_2Estrict__linear__order *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eproc}$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t\in 2.(False\Rightarrow(p\ V0t))) \tag{7}$$

Assume the following.

$$(\forall V0t\in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0y \in A\_27a.(\forall V1P \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0y)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V2x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V2x)\ (ap\ V1P\ V2x)))))) \Leftrightarrow (p\ (ap\ V1P\ V0y))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2P \in ((2^{A\_27b})^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2))\ (\lambda V3x \in A\_27a.(\lambda V4y \in A\_27b.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V3x)\ V4y))\ (ap\ (ap\ V2P\ V3x)\ V4y)))))))) \Leftrightarrow (p\ (ap\ (ap\ V2P\ V0x)\ V1y)))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{12}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V2s\_27 \in (2^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict\_\_linear\_\_order}\ A\_27a)\ V1r)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict\_\_linear\_\_order}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V1r)\ V2s\_27))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V2s\_27)))))))$$