

thm_2Eset_relation_2Esubset_tc (TMdYYpM-
cpTi4x65XqBA5SfKVRzA65GBYfGH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A) P)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_3D (2^{A_27a \cup A_27b}))$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 12 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota. (\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})).$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (3)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((\forall V1p \in \\ (ty_2Epair_2Eprod\ A_27a\ A_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ A_27a. (\forall V3p_2 \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ A_27a\ A_27b)\ V2p_1)\ V3p_2)))))))))) \quad (4) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\ A_27a)\ V1x)\ V2y))\ V0r)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V1x)\ V2y))\ (ap \\ (c_2Eset_relation_2Etc\ A_27a)\ V0r)))))) \wedge (\forall V3x \in A_27a. \\ (\forall V4y \in A_27a. ((\exists V5z \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\ A_27a)\ V3x)\ V5z))\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \wedge (\\ p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V5z)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\ A_27a)\ V0r)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a \\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V3x)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\ A_27a)\ V0r)))))) \quad (5) \end{aligned}$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ V0r)\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r)))) \end{aligned}$$