

thm_2Eset_relation_2Etc_cases
(TMQ4x5p52B2QLQvky4pKyW6kcADXs7htXqB)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) \text{c_2Ebool_2E_2F}))$

Definition 7 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V0v \in 2. \text{c_2Ebool_2E_2T})$.

Definition 8 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } (V1f V0x))))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \quad (1)$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epair_2EABS_prod } A_27a A_27b \in ((\text{ty_2Epair_2Eprod } A_27a A_27b))^{((2^{A_27b})^{A_27a})} \quad (2)$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (\text{ap } (\text{c_2Epair_2EABS_prod } A_27a A_27b))$

Definition 11 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P x) \text{ then } (\lambda x. x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 12 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define `c_2Eset_relation_2Etc` to be $\lambda A_27a : \iota. (\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).$

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \tag{5}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{7}$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in 2. (((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))) \tag{8}$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1y \in 2. (\forall V2z \in 2. (\forall V3w \in 2. (((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))) \tag{10}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \tag{11}$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\ & \quad A_27b.(((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\ & \quad (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in \\ & A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \end{aligned} \quad (14)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)}). \\ & \quad (\forall V1x \in A_27a.(\forall V2y \in A_27a.((p (ap (ap (c_2Ebool_2EIN \\ & \quad (ty_2Epair_2Eprod A_27a A_27a)) (ap (ap (c_2Epair_2E_2C A_27a \\ & \quad A_27a) V1x) V2y)) (ap (c_2Eset_relation_2Etc A_27a) V0r))) \Leftrightarrow (\\ & \quad (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A_27a A_27a)) (ap (\\ & \quad ap (c_2Epair_2E_2C A_27a A_27a) V1x) V2y)) V0r)) \vee (\exists V3z \in \\ & \quad A_27a.((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A_27a A_27a)) \\ & \quad (ap (ap (c_2Epair_2E_2C A_27a A_27a) V1x) V3z)) (ap (c_2Eset_relation_2Etc \\ & \quad A_27a) V0r)))) \wedge (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A_27a \\ & \quad A_27a)) (ap (ap (c_2Epair_2E_2C A_27a A_27a) V3z) V2y)) (ap (c_2Eset_relation_2Etc \\ & \quad A_27a) V0r)))))))))) \end{aligned}$$