

thm_2Eset__relation_2Etc__empty
(TMJ5sUzfAZBtSGM9LVMYe9An9hSBjynTwb2)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 7 We define `c_2Epred__set_2EEMPTY` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. \text{c_2Ebool_2E_2F})$.

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) \text{c_2Ebool_2E_2F}))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \tag{1}$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a A. 27b))^{((2^{A-27b})^{A-27a})} \tag{2}$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap } (\text{c_2Ebool_2E_7E } (V0x = V1y)))$

Definition 11 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota. (\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x))$

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \tag{8}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{9}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg (\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg (p\ (ap\ V0P\ V2x)))))) \tag{10}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B)))))) \tag{11}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x) (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ (\forall V1tc_27 \in ((2^{A_27a})^{A_27a}). ((\forall V2x \in A_27a. (\forall V3y \in \\ A_27a. ((p (ap (ap (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p (ap (ap \\ V1tc_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A_27a. (\forall V5y \in A_27a. \\ ((\exists V6z \in A_27a. ((p (ap (ap\ V1tc_27\ V4x)\ V6z)) \wedge (p (ap (ap\ V1tc_27 \\ V6z)\ V5y)))) \Rightarrow (p (ap (ap\ V1tc_27\ V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A_27a. \\ (\forall V8y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A_27a\ A_27a))\ (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27a)\ V7x)\ V8y))\ (ap \\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \Rightarrow (p (ap (ap\ V1tc_27\ V7x) \\ V8y)))))))))) \end{aligned} \quad (13)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27a)\ V0x)\ V1y))\ (ap (c_2Eset_relation_2Etc \\ A_27a)\ (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_27a\ A_27a))))))))))$$