

thm\_2Eset\_\_relation\_2Etc\_\_ind  
(TMcjtA63ZLh75Pm2x7oUPUgV7gRcUxaE7tK)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{2}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{3}$$

**Definition 7** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b})$

**Definition 8** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (4)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x\ V1y)$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ V0P)))$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.(ap\ (c\_2Emin\_2E\_40\ 2)\ V2t\ V1t2))))$

**Definition 14** We define  $c\_2Eset\_relation\_2Etc$  to be  $\lambda A\_27a : \iota.(\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})).$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (9)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (10)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))) \quad (11)$$

Assume the following.

$$2.(((p \ V0x) \Rightarrow (p \ V1y)) \wedge ((p \ V2z) \Rightarrow (p \ V3w))) \Rightarrow (((p \ V0x) \vee (p \ V2z)) \Rightarrow ((p \ V1y) \vee (p \ V3w)))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p \ (ap \ V0P \ V2x)) \Rightarrow (p \ (ap \ V1Q \ V2x)))) \Rightarrow ((\exists V3x \in A.27a. (p \ (ap \ V0P \ V3x))) \Rightarrow (\exists V4x \in A.27a. (p \ (ap \ V1Q \ V4x)))))))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c.nonempty \ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}). (\forall V1x \in A.27a. (\forall V2y \in A.27b. ((ap \ (ap \ (c.2Epair.2EUNCURRY \ A.27a \ A.27b \ A.27c) \ V0f) \ (ap \ (ap \ (c.2Epair.2E.2C \ A.27a \ A.27b) \ V1x) \ V2y))) = (ap \ (ap \ V0f \ V1x) \ V2y)))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c.nonempty \ A.27c \Rightarrow (\forall V0P \in (A.27c^{(ty.2Epair.2Eprod \ A.27a \ A.27b)}). ((\lambda V1p \in (ty.2Epair.2Eprod \ A.27a \ A.27b). (ap \ V0P \ V1p)) = (ap \ (c.2Epair.2EUNCURRY \ A.27a \ A.27b \ A.27c) \ (\lambda V2p1 \in A.27a. (\lambda V3p2 \in A.27b. (ap \ V0P \ (ap \ (ap \ (c.2Epair.2E.2C \ A.27a \ A.27b) \ V2p1) \ V3p2))))))) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\forall V2y \in A.27b. (p \ (ap \ (ap \ V0P \ V1x) \ V2y))) \Leftrightarrow (p \ (ap \ (c.2Ebool.2E.21 \ (ty.2Epair.2Eprod \ A.27a \ A.27b)) \ (ap \ (c.2Epair.2EUNCURRY \ A.27a \ A.27b \ 2) \ (\lambda V3x \in A.27a. (\lambda V4y \in A.27b. (ap \ (ap \ V0P \ V3x) \ V4y)))))))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1x \in A.27a. ((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V1x) \ V0P)) \Leftrightarrow (p \ (ap \ V0P \ V1x)))) \quad (17)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). \\ (\forall V1tc\_27 \in ((2^{A\_27a})^{A\_27a}). ((\forall V2x \in A\_27a. (\forall V3y \in \\ A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)) \\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p\ (ap\ (ap \\ V1tc\_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A\_27a. (\forall V5y \in A\_27a. \\ ((\exists V6z \in A\_27a. ((p\ (ap\ (ap\ V1tc\_27\ V4x)\ V6z)) \wedge (p\ (ap\ (ap\ V1tc\_27 \\ V6z)\ V5y)))) \Rightarrow (p\ (ap\ (ap\ V1tc\_27\ V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A\_27a. \\ (\forall V8y \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Epair\_2Eprod \\ A\_27a\ A\_27a))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V7x)\ V8y))\ (ap \\ (c\_2Eset\_relation\_2Etc\ A\_27a)\ V0r))) \Rightarrow (p\ (ap\ (ap\ V1tc\_27\ V7x) \\ V8y)))))))))) \end{aligned}$$