

thm_2Eset__relation_2Etc__strongind__right
(TMTMD-
wZqMhk7JgGZsy3mhhL7ti2jzF1nVRL)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0 x \in 2. V0 x)) (\lambda V1 x \in 2. V1 x)$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0 P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))) P))$

Definition 5 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0 t \in 2. V0 t)$.

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0 t1 \in 2. (\lambda V1 t2 \in 2. (ap (c_2Ebool_2E_21 2)) (\lambda V2 t \in 2. V2 t)))$

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0 t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0 t) c_2Ebool_2E_2F))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0 t1 \in 2. (\lambda V1 t2 \in 2. (ap (c_2Ebool_2E_21 2)) (\lambda V2 t \in 2. V2 t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow c_2Epair_2EABS_prod A. 27a A. 27b \in ((ty_2Epair_2Eprod A. 27a A. 27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0 x \in A. 27a. \lambda V1 y \in A. 27b. (ap (c_2Epair_2E_2C A. 27a A. 27b) (V0 x V1 y))$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota.(\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{6}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{10}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{11}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ((p V1B) \wedge (p V2C)))))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)))) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3)))))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (17)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (28) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad A_27a)\ V1x)\ V2y))\ V0r)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V1x)\ V2y))\ (ap \\
& \quad (c_2Eset_relation_2Etc\ A_27a)\ V0r)))))) \wedge (\forall V3x \in A_27a. \\
& \quad (\forall V4y \in A_27a. ((\exists V5z \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\
& \quad A_27a)\ V3x)\ V5z))\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \wedge (\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27a)\ V5z)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\
& \quad A_27a)\ V0r)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V3x)\ V4y))\ (ap\ (c_2Eset_relation_2Etc \\
& \quad A_27a)\ V0r))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad (\forall V1tc_27 \in ((2^{A_27a})^{A_27a}). (((\forall V2x \in A_27a. (\forall V3y \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p\ (ap\ (ap \\
& \quad V1tc_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A_27a. (\forall V5y \in A_27a. \\
& \quad ((\exists V6z \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V4x)\ V6z))\ (ap \\
& \quad (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \wedge ((p\ (ap\ (ap\ V1tc_27\ V4x) \\
& \quad V6z)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V6z)\ V5y))\ (ap\ (c_2Eset_relation_2Etc \\
& \quad A_27a)\ V0r))) \wedge (p\ (ap\ (ap\ V1tc_27\ V6z)\ V5y)))))) \Rightarrow (p\ (ap\ (ap\ V1tc_27 \\
& \quad V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A_27a. (\forall V8y \in A_27a. ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ A_27a)\ V7x)\ V8y))\ (ap\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ V1tc_27\ V7x)\ V8y))))))
\end{aligned} \tag{32}$$

Theorem 1

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ (\forall V1tc_27 \in ((2^{A_27a})^{A_27a}). ((\forall V2x \in A_27a. (\forall V3y \in \\ A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p\ (ap\ (ap \\ V1tc_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A_27a. (\forall V5y \in A_27a. \\ ((\exists V6z \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V4x)\ V6z))\ (ap \\ (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \wedge ((p\ (ap\ (ap\ V1tc_27\ V4x) \\ V6z)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V6z)\ V5y))\ V0r)))))) \Rightarrow (p\ (ap \\ (ap\ V1tc_27\ V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A_27a. (\forall V8y \in A_27a. \\ ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap \\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V7x)\ V8y))\ (ap\ (c_2Eset_relation_2Etc \\ A_27a)\ V0r))) \Rightarrow (p\ (ap\ (ap\ V1tc_27\ V7x)\ V8y))))))))) \end{aligned}$$