

thm_2Eset_relation_2Etc_to_rel_conv (TM- PZp22D1zfC94DHP5GTr82zbQKhKJtRK5m)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Erelation_2E_2TC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_7E$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota.(\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}).$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 15 We define $c_2Eset_relation_2Ereln_to_rel$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2E$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 17 We define $c_2Eset_relation_2Erel_to_reln$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27a$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\
& (2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A.27a.(p \ (\\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A.27a}).((p \ V0P) \wedge (\forall V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\
& A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap \ (c.2Epair_2EFST \ A.27a \\
& A.27b) \ (ap \ (ap \ (c.2Epair_2E_2C \ A.27a \ A.27b) \ V0x) \ V1y)) = V0x)))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap \ (c.2Epair_2ESND \ A.27a \\
& A.27b) \ (ap \ (ap \ (c.2Epair_2E_2C \ A.27a \ A.27b) \ V0x) \ V1y)) = V1y)))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\
& ((\forall V1x \in A.27a.(\forall V2y \in A.27a.((p \ (ap \ (ap \ V0R \ V1x) \ V2y)) \Rightarrow \\
& (p \ (ap \ (ap \ (ap \ (c.2Erelation_2ETC \ A.27a) \ V0R) \ V1x) \ V2y)))))) \wedge (\forall V3x \in \\
& A.27a.(\forall V4y \in A.27a.(\forall V5z \in A.27a.(((p \ (ap \ (ap \ (ap \\
& (c.2Erelation_2ETC \ A.27a) \ V0R) \ V3x) \ V4y)) \wedge (p \ (ap \ (ap \ (ap \ (c.2Erelation_2ETC \\
& A.27a) \ V0R) \ V4y) \ V5z))) \Rightarrow (p \ (ap \ (ap \ (ap \ (c.2Erelation_2ETC \ A.27a) \\
& V0R) \ V3x) \ V5z))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1P \in ((2^{A_27a})^{A_27a}). (((\forall V2x \in A_27a. (\forall V3y \in \\
& A_27a. ((p (ap (ap (ap V0R V2x) V3y)) \Rightarrow (p (ap (ap V1P V2x) V3y)))))) \wedge (\forall V4x \in \\
& A_27a. (\forall V5y \in A_27a. (\forall V6z \in A_27a. (((p (ap (ap V1P \\
& V4x) V5y)) \wedge (p (ap (ap V1P V5y) V6z))) \Rightarrow (p (ap (ap V1P V4x) V6z)))))) \Rightarrow \\
& (\forall V7u \in A_27a. (\forall V8v \in A_27a. ((p (ap (ap (ap (c_2Erelation_2ETC \\
& A_27a) V0R) V7u) V8v)) \Rightarrow (p (ap (ap V1P V7u) V8v)))))))))
\end{aligned} \tag{18}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{22}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r)))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0s \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). (\forall V1t \in \\ & \quad (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in \\ & \quad A_27a. (\forall V3y \in A_27b. ((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod \\ & \quad A_27a\ A_27b)) (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27b) V2x) V3y)) V0s)) \Leftrightarrow \\ & \quad (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod\ A_27a\ A_27b)) (ap (\\ & \quad ap (c_2Epair_2E_2C\ A_27a\ A_27b) V2x) V3y)) V1t))))))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & \quad ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27a)) (ap (ap (c_2Epair_2E_2C\ A_27a \\ & \quad A_27a) V1x) V2y)) V0r)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod \\ & \quad A_27a\ A_27a)) (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27a) V1x) V2y)) (ap \\ & \quad (c_2Eset_relation_2Etc\ A_27a) V0r)))))) \wedge (\forall V3x \in A_27a. \\ & \quad (\forall V4y \in A_27a. ((\exists V5z \in A_27a. ((p (ap (ap (c_2Ebool_2EIN \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27a)) (ap (ap (c_2Epair_2E_2C\ A_27a \\ & \quad A_27a) V3x) V5z)) (ap (c_2Eset_relation_2Etc\ A_27a) V0r))) \wedge (\\ & \quad p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod\ A_27a\ A_27a)) (ap (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27a) V5z) V4y)) (ap (c_2Eset_relation_2Etc \\ & \quad A_27a) V0r)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod\ A_27a \\ & \quad A_27a)) (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27a) V3x) V4y)) (ap (c_2Eset_relation_2Etc \\ & \quad A_27a) V0r))))))))) \quad (32) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}), \\
& \quad (\forall V1tc_27 \in ((2^{A_27a})^{A_27a}).((\forall V2x \in A_27a.(\forall V3y \in \\
& \quad A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2x)\ V3y))\ V0r)) \Rightarrow (p\ (ap\ (ap \\
& \quad V1tc_27\ V2x)\ V3y)))))) \wedge (\forall V4x \in A_27a.(\forall V5y \in A_27a. \\
& \quad ((\exists V6z \in A_27a.((p\ (ap\ (ap\ V1tc_27\ V4x)\ V6z)) \wedge (p\ (ap\ (ap\ V1tc_27 \\
& \quad V6z)\ V5y)))) \Rightarrow (p\ (ap\ (ap\ V1tc_27\ V4x)\ V5y)))))) \Rightarrow (\forall V7x \in A_27a. \\
& \quad (\forall V8y \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V7x)\ V8y))\ (ap \\
& \quad (c_2Eset_relation_2Etc\ A_27a)\ V0r))) \Rightarrow (p\ (ap\ (ap\ V1tc_27\ V7x) \\
& \quad V8y)))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0xy \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\forall V1R \in ((\\
& \quad 2^{A_27b})^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b))\ V0xy)\ (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a \\
& \quad A_27b)\ V1R))) \Leftrightarrow (p\ (ap\ (ap\ V1R\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0xy)) \\
& \quad (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0xy))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).(\forall V1x \in \\
& \quad A_27a.(\forall V2y \in A_27b.((p\ (ap\ (ap\ (ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad A_27a\ A_27b)\ V0r)\ V1x)\ V2y)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))\ V0r))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27b})^{A_27a}).((ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Erel_to_reln\ A_27a\ A_27b) \\
& \quad V0R)) = V0R))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).((ap\ (c_2Eset_relation_2Erel_to_reln \\
& \quad A_27a\ A_27b)\ (ap\ (c_2Eset_relation_2Ereln_to_rel\ A_27a\ A_27b) \\
& \quad V0r)) = V0r))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0r1 \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}), (\forall V1r2 \in \\
& (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}), (((ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad A.27a\ A.27b)\ V0r1) = (ap\ (c_2Eset_relation_2Ereln_to_rel\ A.27a \\
& \quad A.27b)\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \quad (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0R1 \in ((2^{A.27b})^{A.27a}), (\forall V1R2 \in ((2^{A.27b})^{A.27a}). \\
& (((ap\ (c_2Eset_relation_2Erel_to_reln\ A.27a\ A.27b)\ V0R1) = \\
& (ap\ (c_2Eset_relation_2Erel_to_reln\ A.27a\ A.27b)\ V1R2)) \Leftrightarrow \\
& \quad (V0R1 = V1R2)))) \quad (39)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)}), \\
& ((ap\ (c_2Eset_relation_2Etc\ A.27a)\ V0r) = (ap\ (c_2Eset_relation_2Erel_to_reln \\
& A.27a\ A.27a)\ (ap\ (c_2Erelation_2ETC\ A.27a)\ (ap\ (c_2Eset_relation_2Ereln_to_rel \\
& \quad A.27a\ A.27a)\ V0r))))))
\end{aligned}$$