

thm_2Eset__relation_2Etc__transitive
(TMJXtTTmuBLXikZy5Bye5ChXCkVDKuFNyr4)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 11 We define `c_2Epair_2E_2C` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 12 We define $c_2Eset_relation_2Etc$ to be $\lambda A_27a : \iota. (\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}))$.

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$.

Definition 14 We define $c_2Eset_relation_2Etransitive$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \tag{5}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{6}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \tag{8}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x))))))) \tag{9}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \tag{10}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (13)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (17)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (18)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})}). \\ & \quad ((\forall V1x \in A_{.27a}. (\forall V2y \in A_{.27a}. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \\ & \quad (ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_{.27a} \\ & \quad A_{.27a} \ V1x \ V2y)) \ V0r)) \Rightarrow (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Epair_2Eprod \\ & \quad A_{.27a} \ A_{.27a})) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_{.27a} \ A_{.27a} \ V1x \ V2y)) \ (ap \\ & \quad (c_2Eset_relation_2Etc \ A_{.27a}) \ V0r)))))) \wedge (\forall V3x \in A_{.27a}. \\ & \quad (\forall V4y \in A_{.27a}. ((\exists V5z \in A_{.27a}. ((p \ (ap \ (ap \ (c_2Ebool_2EIN \\ & \quad (ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_{.27a} \\ & \quad A_{.27a} \ V3x \ V5z)) \ (ap \ (c_2Eset_relation_2Etc \ A_{.27a}) \ V0r))) \wedge (\\ & \quad p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})) \ (ap \ (ap \\ & \quad (c_2Epair_2E_2C \ A_{.27a} \ A_{.27a} \ V5z \ V4y)) \ (ap \ (c_2Eset_relation_2Etc \\ & \quad A_{.27a}) \ V0r)))))) \Rightarrow (p \ (ap \ (ap \ (c_2Ebool_2EIN \ (ty_2Epair_2Eprod \ A_{.27a} \\ & \quad A_{.27a})) \ (ap \ (ap \ (c_2Epair_2E_2C \ A_{.27a} \ A_{.27a} \ V3x \ V4y)) \ (ap \ (c_2Eset_relation_2Etc \\ & \quad A_{.27a}) \ V0r)))))) \end{aligned} \quad (24)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})}). \\ & \quad (p \ (ap \ (c_2Eset_relation_2Etransitive \ A_{.27a}) \ (ap \ (c_2Eset_relation_2Etc \\ & \quad A_{.27a}) \ V0r)))) \end{aligned}$$