

# thm\_2Esorting\_2EALL\_\_DISTINCT\_\_PERM (TMU27F8hrnaweuJFxPofuwvy6LLUz8zjGZi)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).)(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a)))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EELIST\_TO\_SET A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (2)$$

**Definition 5** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).)(ap V1f V0x))$

Let  $c\_2Elist\_2EALL\_DISTINCT : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EALL\_DISTINCT A\_27a \in (2^{(ty\_2Elist\_2Elist A\_27a)}) \quad (3)$$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).)(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 7** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EFILTER A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))^{(2^{A\_27a})}) \quad (4)$$

**Definition 12** We define  $c\_2Esorting\_2E\_PERM$  to be  $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist A\_27a).\lambda V1L2$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))^{(ty\_2Elist\_2Elist A\_27a)}) \quad (5)$$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in (2^{A\_27a}). ((\exists V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a. (p\ (ap\ V1Q\ V4x))))))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\exists V2x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3)))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \quad (19)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0e \in A\_27a. (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l1))) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V2l2)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (c\_2Elist\_2EALL\_DISTINCT \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))) \Leftrightarrow ((p\ (ap \\ & (c\_2Elist\_2EALL\_DISTINCT\ A\_27a)\ V0l1)) \wedge ((p\ (ap\ (c\_2Elist\_2EALL\_DISTINCT \\ & A\_27a)\ V1l2)) \wedge (\forall V2e \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V2e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V0l1)))) \Rightarrow (\neg(p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V2e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a) \\ & V1l2)))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg( \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (A\_27b^{(ty\_2Elist\_2Elist \ A\_27a)}). ((\forall V1x1 \in \\
& \quad (ty\_2Elist\_2Elist \ A\_27a). (\forall V2x2 \in (ty\_2Elist\_2Elist \ A\_27a). \\
& \quad (\forall V3x3 \in (ty\_2Elist\_2Elist \ A\_27a). ((ap \ V0f \ (ap \ (ap \ (c\_2Elist\_2EAPPEND \\
& \quad A\_27a) \ (ap \ (ap \ (c\_2Elist\_2EAPPEND \ A\_27a) \ V1x1) \ V2x2)) \ V3x3)) = ( \\
& \quad ap \ V0f \ (ap \ (ap \ (c\_2Elist\_2EAPPEND \ A\_27a) \ (ap \ (ap \ (c\_2Elist\_2EAPPEND \\
& \quad A\_27a) \ V1x1) \ V3x3)) \ V2x2)))))) \Rightarrow (\forall V4x \in (ty\_2Elist\_2Elist \\
& \quad A\_27a). (\forall V5y \in (ty\_2Elist\_2Elist \ A\_27a). ((p \ (ap \ (ap \ (c\_2Esorting\_2Eperm \\
& \quad A\_27a) \ V4x) \ V5y)) \Rightarrow ((ap \ V0f \ V4x) = (ap \ V0f \ V5y)))))) \quad (33)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist \ A\_27a). ((p \ (ap \ (ap \ (c\_2Esorting\_2Eperm \\
& \quad A\_27a) \ V0l1) \ V1l2)) \Rightarrow ((p \ (ap \ (c\_2Elist\_2EALL\_DISTINCT \ A\_27a) \\
& \quad V0l1)) \Leftrightarrow (p \ (ap \ (c\_2Elist\_2EALL\_DISTINCT \ A\_27a) \ V1l2))))))
\end{aligned}$$