

thm_2Esorting_2EFILTER_EQ_LENGTHS_EQ (TMHGseHk5f9wtevLTScffphz1az7m9Mxxar)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (2)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (3)$$

Let $c_2Erich_list_2EREPLICATE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EREPLICATE\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{A_27a})^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (5)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in \\ & \quad (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\ & \quad (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x))\ V1l) = (ap\ (ap\ (c_2Erich_list_2EREPLICATE \\ & \quad A_27a)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER \\ & \quad A_27a)\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x))\ V1l)))\ V0x)))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l1 \in \\ & \quad (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist\ A_27a). \\ & \quad (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\ & \quad (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x))\ V1l1)) = (ap\ (c_2Elist_2ELENGTH \\ & \quad A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D\ A_27a) \\ & \quad V0x))\ V2l2))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D \\ & \quad A_27a)\ V0x))\ V1l1) = (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D \\ & \quad A_27a)\ V0x))\ V2l2)))))) \end{aligned}$$