

thm_2Esorting_2EFOLDR__PERM (TMdb6oB2CmKGHhDjKDEjyJbA5LmPm9JqnLQ)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A.27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } A.27a))))$

Definition 5 We define `c_2Ecombin_2EK` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V 0x \in A. 27a. (\lambda V 1y \in A. 27b. V 0x))$

Definition 6 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V 0f \in ((A. 27c^{A.27b})^{A.27a}))$

Definition 7 We define `c_2Ecombin_2EI` to be $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. 27a (A. 27a^{A.27a})) A. 27a))$

Definition 8 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A.27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A.27a}))))$

Definition 9 We define `c_2Ecombin_2EASSOC` to be $\lambda A. 27a : \iota. \lambda V 0f \in ((A. 27a^{A.27a})^{A.27a}). (\text{ap } (\text{c_2Ebool_2E_21 } A. 27a))$

Definition 10 We define `c_2Ecombin_2ECOMM` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V 0f \in ((A. 27b^{A.27a})^{A.27a}). (\text{ap } (\text{c_2Ebool_2E_21 } A. 27a))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A 0) \quad (1)$$

Let `c_2Elist_2EFOLDR` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Elist_2EFOLDR } A. 27a A. 27b \in (((A. 27b^{(\text{ty_2Elist_2Elist } A. 27a)})^{A. 27b})^{((A. 27b^{A. 27b})^{A. 27a}))} \quad (2)$$

Definition 11 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2. V 0t))$.

Definition 12 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (3)$$

Definition 15 We define $c_2Esorting_2Eperm$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist A_27a).\lambda V1L2$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (5)$$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in \\
& A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).(((p \ V0P) \wedge (\forall V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \Rightarrow (31)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}^{A_{27a}}})^{A_{27a}}).(\forall V1x \in A_{27a}.(\exists V2y \in A_{27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}).(\forall V4x \in A_{27a}.(p (ap (ap V0P V4x) (ap V3f V4x))))))) (32)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_{2Ecombin}_{2El} A_{27a}) V0x) = V0x)) (33)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0f \in ((A_{27b}^{A_{27a}})^{A_{27a}}).(\forall V1e \in A_{27b}.((ap (ap (ap (c_{2Elist}_{2EFOLDR} A_{27a} A_{27b}) V0f) V1e) (c_{2Elist}_{2ENIL} A_{27a})) = V1e))) \wedge (\forall V2f \in ((A_{27b}^{A_{27a}})^{A_{27a}}).(\forall V3e \in A_{27b}.(\forall V4x \in A_{27a}.(\forall V5l \in (ty_{2Elist}_{2Elist} A_{27a}).((ap (ap (ap (c_{2Elist}_{2EFOLDR} A_{27a} A_{27b}) V2f) V3e) (ap (ap (c_{2Elist}_{2ECONS} A_{27a} V4x) V5l)) = (ap (ap V2f V4x) (ap (ap (ap (c_{2Elist}_{2EFOLDR} A_{27a} A_{27b}) V2f) V3e) V5l)))))))))) (34)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) (35)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) (38)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in ((2^{(ty_2Elist_2Elist A.27a)})(ty_2Elist_2Elist A.27a)), \\
& (((p (ap V0P (c.2Elist_2ENIL A.27a)) (c.2Elist_2ENIL A.27a))) \wedge \\
& ((\forall V1x \in A.27a. (\forall V2l1 \in (ty_2Elist_2Elist A.27a). \\
& (\forall V3l2 \in (ty_2Elist_2Elist A.27a). ((p (ap (ap V0P V2l1) V3l2)) \Rightarrow \\
& (p (ap (ap V0P (ap (ap (c.2Elist_2ECONS A.27a) V1x) V2l1)) (ap (ap \\
& (c.2Elist_2ECONS A.27a) V1x) V3l2)))))) \wedge ((\forall V4x \in A.27a. \\
& (\forall V5y \in A.27a. (\forall V6l1 \in (ty_2Elist_2Elist A.27a). \\
& (\forall V7l2 \in (ty_2Elist_2Elist A.27a). ((p (ap (ap V0P V6l1) V7l2)) \Rightarrow \\
& (p (ap (ap V0P (ap (ap (c.2Elist_2ECONS A.27a) V4x) (ap (ap (c.2Elist_2ECONS \\
& A.27a) V5y) V6l1)) (ap (ap (c.2Elist_2ECONS A.27a) V5y) (ap (ap \\
& (c.2Elist_2ECONS A.27a) V4x) V7l2)))))) \wedge ((\forall V8l1 \in (ty_2Elist_2Elist \\
& A.27a). (\forall V9l2 \in (ty_2Elist_2Elist A.27a). (\forall V10l3 \in \\
& (ty_2Elist_2Elist A.27a). (((p (ap (ap V0P V8l1) V9l2)) \wedge (p (ap (\\
& ap V0P V9l2) V10l3))) \Rightarrow (p (ap (ap V0P V8l1) V10l3)))))) \Rightarrow (\forall V11l1 \in \\
& (ty_2Elist_2Elist A.27a). (\forall V12l2 \in (ty_2Elist_2Elist \\
& A.27a). ((p (ap (ap (c.2Esortng_2EPERM A.27a) V11l1) V12l2)) \Rightarrow \\
& (p (ap (ap V0P V11l1) V12l2))))))
\end{aligned} \tag{45}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}). \\ & (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist \\ & A_27a). (\forall V3e \in A_27a. (((p\ (ap\ (c_2Ecombin_2EASSOC\ A_27a) \\ & V0f)) \wedge (p\ (ap\ (c_2Ecombin_2ECOMM\ A_27a\ A_27a)\ V0f))) \Rightarrow ((p\ (ap\ (ap \\ & (c_2Esorting_2EPERM\ A_27a)\ V1l1)\ V2l2)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDR \\ & A_27a\ A_27a)\ V0f)\ V3e)\ V1l1) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDR\ A_27a \\ & A_27a)\ V0f)\ V3e)\ V2l2)))))))))) \end{aligned}$$