

thm_2Esorting_2EPERM_CONG_APPEND_IFF (TMLV8HWCEjCTZtkUT6YhgUcCWTAWYpkLKc8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(ty_2Elist_2Elist A_27a)) \quad (2)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(2^{A_27a})) \quad (3)$$

Definition 9 We define $c_2\text{Esorting_2EPERM}$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist A_27a).\lambda V1L2 \in$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))) \tag{9}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x)))))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{11}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{12}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{13}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (16)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (17)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (18)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (23)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ & \quad A.27a).(\forall V1l1 \in (ty_2Elist_2Elist A.27a).(\forall V2l2 \in \\ & \quad (ty_2Elist_2Elist A.27a).((p (ap (ap (c_2Esorting_2Eperm A.27a) \\ & (ap (ap (c_2Elist_2Eappend A.27a) V0l) V1l1)) (ap (ap (c_2Elist_2Eappend \\ & A.27a) V0l) V2l2))) \Leftrightarrow (p (ap (ap (c_2Esorting_2Eperm A.27a) V1l1) \\ & \quad V2l2)))))) \wedge (\forall V3l \in (ty_2Elist_2Elist A.27a).(\forall V4l1 \in \\ & \quad (ty_2Elist_2Elist A.27a).(\forall V5l2 \in (ty_2Elist_2Elist A.27a). \\ & \quad ((p (ap (ap (c_2Esorting_2Eperm A.27a) (ap (ap (c_2Elist_2Eappend \\ & A.27a) V4l1) V3l)) (ap (ap (c_2Elist_2Eappend A.27a) V5l2) V3l))) \Leftrightarrow \\ & \quad (p (ap (ap (c_2Esorting_2Eperm A.27a) V4l1) V5l2))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\ & \quad A.27a).(\forall V1y \in (ty_2Elist_2Elist A.27a).((p (ap (ap (c_2Esorting_2Eperm \\ & A.27a) V0x) V1y)) \Leftrightarrow ((ap (c_2Esorting_2Eperm A.27a) V0x) = (ap (c_2Esorting_2Eperm \\ & \quad A.27a) V1y)))))) \end{aligned} \quad (27)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & \quad A.27a).(\forall V1l1 \in (ty_2Elist_2Elist A.27a).(\forall V2l1_27 \in \\ & \quad (ty_2Elist_2Elist A.27a).(\forall V3l2 \in (ty_2Elist_2Elist A.27a). \\ & \quad (\forall V4l2_27 \in (ty_2Elist_2Elist A.27a).((p (ap (ap (c_2Esorting_2Eperm \\ & A.27a) V1l1) (ap (ap (c_2Elist_2Eappend A.27a) V0l) V2l1_27))) \Rightarrow \\ & \quad ((p (ap (ap (c_2Esorting_2Eperm A.27a) V3l2) (ap (ap (c_2Elist_2Eappend \\ & A.27a) V0l) V4l2_27))) \Rightarrow ((p (ap (ap (c_2Esorting_2Eperm A.27a) \\ & V1l1) V3l2)) \Leftrightarrow (p (ap (ap (c_2Esorting_2Eperm A.27a) V2l1_27) V4l2_27)))))))))) \end{aligned}$$