

# thm\_2Esorting\_2EPERM\_\_FUN\_\_APPEND\_\_IFF (TMPMNNhr5hAQryF22u1pcT3ogABsgHX18Bg)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2Eappend : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2Eappend A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(ty\_2Elist\_2Elist A\_27a)) \quad (2)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Elist\_2Efilter : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2Efilter A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(2^{A\_27a})) \quad (3)$$

**Definition 8** We define  $c\_2Esorting\_2EPERM$  to be  $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist A\_27a).\lambda V1L2 \in (ty\_2Elist\_2Elist A\_27a)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V2l2 \in \\ & (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a) \\ & (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ V0l)\ V2l2))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V1l1) \\ & V2l2)))))) \wedge (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V4l1 \in \\ & (ty\_2Elist\_2Elist\ A\_27a). (\forall V5l2 \in (ty\_2Elist\_2Elist\ A\_27a). \\ & ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ V4l1)\ V3l))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5l2)\ V3l))) \Leftrightarrow \\ & (p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V4l1)\ V5l2)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1y \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM \\ & A\_27a)\ V0x)\ V1y)) \Leftrightarrow ((ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V0x) = (ap\ (c\_2Esorting\_2EPERM \\ & A\_27a)\ V1y)))) \end{aligned} \quad (9)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V2l2 \in \\ & (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V1l1) = \\ & (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V2l2)) \Rightarrow ((ap\ (c\_2Esorting\_2EPERM \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V1l1)) = (ap\ (c\_2Esorting\_2EPERM \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V2l2)))))) \end{aligned}$$