thm_2Esorting_2EPERM__FUN__CONS__IFF (TMY5yrkFpwpLsonAEnyHxUXKJ9qW6ZKnFHM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{2}$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{3}$$

Definition 8 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2 \in$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1l2))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V2l1)\ V1l2)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1y \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x)\ V1y)) \Leftrightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1y))))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V2l2)) \Rightarrow ((ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1l1)) = (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2l2)))))))$$