

thm_2Esorting_2EPERM_FUN_SPLIT (TMUK-
BvWq4aFbFrQXZcoMh3kAKG5LfiWFgnu)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_21` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define `c_2Ebool_2E_21` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `c_2Elist_2EAPPEND` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(ty_2Elist_2Elist A_27a)) \quad (2)$$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let `c_2Elist_2EFILTER` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(2^{A_27a})) \quad (3)$$

Definition 9 We define `c_2Esorting_2EPERM` to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist A_27a).\lambda V1L2 \in (ty_2Elist_2Elist A_27a)$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ & (2^{A_27a}). ((\forall V2x \in A_27a. ((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a. (p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A_27a. (p \ (\\ & ap \ V1Q \ V4x)))))) \end{aligned} \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{19}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{20}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{21}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{22}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{23}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_27a).(\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l2 \in \\
& \quad (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a) \\
& (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l)\ V1l1))\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V0l)\ V2l2))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V1l1) \\
& \quad V2l2)))))) \wedge (\forall V3l \in (ty_2Elist_2Elist\ A_27a).(\forall V4l1 \in \\
& \quad (ty_2Elist_2Elist\ A_27a).(\forall V5l2 \in (ty_2Elist_2Elist\ A_27a). \\
& \quad ((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V4l1)\ V3l))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V5l2)\ V3l))) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V4l1)\ V5l2)))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& \quad A_27a).(\forall V1y \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Esorting_2EPERM \\
& \quad A_27a)\ V0x)\ V1y)) \Leftrightarrow ((ap\ (c_2Esorting_2EPERM\ A_27a)\ V0x) = (ap\ (c_2Esorting_2EPERM \\
& \quad A_27a)\ V1y))))))
\end{aligned} \tag{26}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& \quad A_27a).(\forall V1l1 \in (ty_2Elist_2Elist\ A_27a).(\forall V2l1_27 \in \\
& \quad (ty_2Elist_2Elist\ A_27a).(\forall V3l2 \in (ty_2Elist_2Elist\ A_27a). \\
& \quad ((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V0l)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V1l1)\ V3l2))) \Rightarrow ((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V2l1_27) \\
& \quad V1l1)) \Rightarrow (p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V0l)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ V2l1_27)\ V3l2))))))))))
\end{aligned}$$