

thm_2Esorting_2Eperm_ind (TMXlfZdwpKHLpJvdsDp63duhifAtskcWyvG)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Emarker_2E_3A_2D$ to be $\lambda V0lab \in \omega.\lambda V1argument \in 2.V1argument$.

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.27a \Rightarrow c_2Elist_2ENIL A.27a \in (ty_2Elist_2Elist A.27a) \quad (2)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.27a \Rightarrow c_2Elist_2EAPPEND A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})^{(ty_2Elist_2Elist A.27a)}) \quad (3)$$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (5)$$

Definition 12 We define $c_2Esorting_2Eperm$ to be $\lambda A_27a : \iota. \lambda V0L1 \in (ty_2Elist_2Elist\ A_27a). \lambda V1L2$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (10)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).(\forall V2x \in A_27a.((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \Rightarrow (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ (p\ V1B)))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in \\ A_27a.((\forall V2x \in A_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\ ap\ V0f\ V1v)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & ((\forall V0l \in (\text{ty_2Elist_2Elist} \\ A_27a).((\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) (\text{c_2Elist_2ENIL } A_27a)) \\ V0l) = V0l)) \wedge (\forall V1l1 \in (\text{ty_2Elist_2Elist } A_27a).(\forall V2l2 \in \\ (\text{ty_2Elist_2Elist } A_27a).(\forall V3h \in A_27a.((\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } \\ A_27a) (\text{ap } (\text{ap } (\text{c_2Elist_2ECONS } A_27a) V3h) V1l1)) V2l2) = (\text{ap } (\text{ap } \\ (\text{c_2Elist_2ECONS } A_27a) V3h) (\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) \\ V1l1) V2l2)))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0P \in (2^{(\text{ty_2Elist_2Elist } A_27a)}). \\ ((\text{p } (\text{ap } V0P (\text{c_2Elist_2ENIL } A_27a))) \wedge (\forall V1t \in (\text{ty_2Elist_2Elist} \\ A_27a).(\text{p } (\text{ap } V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(\text{p } (\text{ap } V0P (\text{ap } (\text{ap } (\\ \text{c_2Elist_2ECONS } A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (\text{ty_2Elist_2Elist} \\ A_27a).(\text{p } (\text{ap } V0P V3l)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0l1 \in (\text{ty_2Elist_2Elist} \\ A_27a).(\forall V1l2 \in (\text{ty_2Elist_2Elist } A_27a).(\forall V2l3 \in \\ (\text{ty_2Elist_2Elist } A_27a).((\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) \\ V0l1) (\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) V1l2) V2l3)) = (\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } \\ A_27a) (\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) V0l1) V1l2)) V2l3)))))) \end{aligned} \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(\text{p } V0t))) \Leftrightarrow (\text{p } V0t))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.((\text{p } V0A) \Rightarrow ((\neg(\text{p } V0A)) \Rightarrow \text{False}))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\text{p } V0A) \vee (\text{p } V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((\text{p } V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(\text{p } V1B)) \Rightarrow \text{False})))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\text{p } V0A)) \vee (\text{p } V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((\text{p } V0A) \Rightarrow ((\neg(\text{p } V1B)) \Rightarrow \text{False})))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(\text{p } V0A)) \Rightarrow \text{False}) \Rightarrow (((\text{p } V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in (ty_2Elist_2Elist \\
& A.27a). (\forall V1y \in (ty_2Elist_2Elist A.27a). (\forall V2z \in \\
& (ty_2Elist_2Elist A.27a). (((p (ap (ap (c_2Esorting_2Eperm A.27a) \\
& V0x) V1y)) \wedge (p (ap (ap (c_2Esorting_2Eperm A.27a) V1y) V2z))) \Rightarrow (\\
& p (ap (ap (c_2Esorting_2Eperm A.27a) V0x) V2z))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1l2 \in \\
& (ty_2Elist_2Elist A.27a). (\forall V2l1 \in (ty_2Elist_2Elist A.27a). \\
& ((p (ap (ap (c_2Esorting_2Eperm A.27a) (ap (ap (c_2Elist_2Econs \\
& A.27a) V0x) V2l1)) (ap (ap (c_2Elist_2Econs A.27a) V0x) V1l2))) \Leftrightarrow \\
& (p (ap (ap (c_2Esorting_2Eperm A.27a) V2l1) V1l2))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0L \in (ty_2Elist_2Elist \\ & A_27a).(((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V0L)\ (c_2Elist_2ENIL \\ & A_27a)))) \Leftrightarrow (V0L = (c_2Elist_2ENIL\ A_27a))) \wedge ((p\ (ap\ (ap\ (c_2Esorting_2EPERM \\ & A_27a)\ (c_2Elist_2ENIL\ A_27a))\ V0L)) \Leftrightarrow (V0L = (c_2Elist_2ENIL\ A_27a)))))) \\ & \hspace{15em} (39) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1L \in (ty_2Elist_2Elist\ A_27a).(\forall V2h \in \\ & A_27a.((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V0t))\ V1L)) \Leftrightarrow (\exists V3M \in (ty_2Elist_2Elist\ A_27a). \\ & (\exists V4N \in (ty_2Elist_2Elist\ A_27a).((V1L = (ap\ (ap\ (c_2Elist_2EAPPEND \\ & A_27a)\ V3M)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V4N)))) \wedge (p\ (ap\ (\\ & ap\ (c_2Esorting_2EPERM\ A_27a)\ V0t)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\ & A_27a)\ V3M)\ V4N)))))))))) \\ & \hspace{15em} (40) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in ((2^{(ty_2Elist_2Elist\ A_27a)})(ty_2Elist_2Elist\ A_27a)). \\ & (((p\ (ap\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))\ (c_2Elist_2ENIL\ A_27a)))) \wedge \\ & ((\forall V1x \in A_27a.(\forall V2l1 \in (ty_2Elist_2Elist\ A_27a). \\ & (\forall V3l2 \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ V0P\ V2l1)\ V3l2)) \Rightarrow \\ & (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1x)\ V2l1))\ (ap\ (ap \\ & (c_2Elist_2ECONS\ A_27a)\ V1x)\ V3l2)))))) \wedge ((\forall V4x \in A_27a. \\ & (\forall V5y \in A_27a.(\forall V6l1 \in (ty_2Elist_2Elist\ A_27a). \\ & (\forall V7l2 \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ V0P\ V6l1)\ V7l2)) \Rightarrow \\ & (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V4x)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V5y)\ V6l1)))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5y)\ (ap\ (ap \\ & (c_2Elist_2ECONS\ A_27a)\ V4x)\ V7l2)))))) \wedge (\forall V8l1 \in (ty_2Elist_2Elist \\ & A_27a).(\forall V9l2 \in (ty_2Elist_2Elist\ A_27a).(\forall V10l3 \in \\ & (ty_2Elist_2Elist\ A_27a).(((p\ (ap\ (ap\ V0P\ V8l1)\ V9l2)) \wedge (p\ (ap\ (\\ & ap\ V0P\ V9l2)\ V10l3))) \Rightarrow (p\ (ap\ (ap\ V0P\ V8l1)\ V10l3)))))) \Rightarrow (\forall V11l1 \in \\ & (ty_2Elist_2Elist\ A_27a).(\forall V12l2 \in (ty_2Elist_2Elist \\ & A_27a).((p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V11l1)\ V12l2)) \Rightarrow \\ & (p\ (ap\ (ap\ V0P\ V11l1)\ V12l2)))))) \end{aligned}$$