thm_2Esorting_2EPERM__LENGTH (TMXfh-NapkVrvFckHrPuR4RA81wFtCSkHwxR)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{5}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{8}$$

Definition 9 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2 \in$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{9}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{10}$$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{15}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0})\wedge(\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}). (((p\ (ap\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))\wedge ((\forall V1x \in A\_27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ V0P\ V2l1)\ V3l2))\Rightarrow (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ V2l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ V3l2)))))))\wedge((\forall V4x \in A\_27a. (\forall V5y \in A\_27a.(\forall V6l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V7l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ V0P\ V6l1)\ V7l2))\Rightarrow (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ V6l1)))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V7l2))))))))))\wedge(\forall V8l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V9l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V10l3 \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ V0P\ V8l1)\ V9l2))\wedge(p\ (ap\ (ap\ V0P\ V9l2)\ V10l3)))\Rightarrow(p\ (ap\ (ap\ V0P\ V8l1)\ V10l3)))))))))\Rightarrow(\forall V11l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V12l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V11l1)\ V12l2))\Rightarrow (p\ (ap\ (ap\ V0P\ V11l1)\ V12l2))))))) \tag{19}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))\Rightarrow((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l2)))))$$