

thm_2Esorting_2EPERM__SET__TO__LIST__count__COUNT__LIST (TMNPKidkoMQogFu5cuTT1aoGPitVsqqZDT8)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (2)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2ECHOICE A_27a \in (A_27a^{(2^{A_27a})}) \quad (3)$$

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (5)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (6)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ x\ s)$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ s\ t)$

Definition 14 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ s\ x))$

Definition 15 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ s\ s))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.t1 \wedge t2)))$

Definition 18 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ s)$

Definition 19 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x \wedge y))$

Definition 20 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 21 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ A_27a)^{A_27a})\ A_27a)$

Definition 22 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 23 We define $c_ERelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_Ebool_2E_21$

Definition 24 We define $c_ERelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1f$

Definition 25 We define $c_ERelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b$

Definition 26 We define $c_ERelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 27 We define $c_ERelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 28 We define $c_ERelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 29 We define $c_Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_ERelation_2EWFREC\ (2^{A_27a})$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{9}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{10}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{11}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{12}$$

Definition 30 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 31 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 32 We define $c_2Epred_set_2Ecount$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (c_2Epred_set_2EG$

Let $c_2Elist_2EGENLIST : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EGENLIST\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Enum_2Enum})^{(A_27a^{ty_2Enum_2Enum})}) \tag{13}$$

Let $c_2Erich_list_2ECOUNT_LIST : \iota$ be given. Assume the following.

$$c_2Erich_list_2ECOUNT_LIST \in ((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{ty_2Enum_2Enum}) \tag{14}$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \tag{15}$$

Definition 33 We define $c_2Esorting_2EPERM$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist\ A_27a).\lambda V1L2$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in \left((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)} \right) \quad (16)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in \left(2^{(ty_2Elist_2Elist\ A_27a)} \right) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (25)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \Rightarrow (26)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \Rightarrow (27)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_{2Ecombin}_{2EI} A_{27a}) V0x) = V0x)) \Rightarrow (28)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).((p (ap (c_{2Epred}_{set}_{2EFINITE} A_{27a}) V0s)) \Rightarrow (\forall V1x \in A_{27a}.((p (ap (ap (c_{2Ebool}_{2EIN} A_{27a}) V1x) (ap (c_{2Elist}_{2ELIST}_{TO}_{SET} A_{27a}) (ap (c_{2Elist}_{2ESET}_{TO}_{LIST} A_{27a}) V0s)))) \Leftrightarrow (p (ap (ap (c_{2Ebool}_{2EIN} A_{27a}) V1x) V0s)))))) \Rightarrow (29)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).((p (ap (c_{2Epred}_{set}_{2EFINITE} A_{27a}) V0s)) \Rightarrow (p (ap (c_{2Elist}_{2EALL}_{DISTINCT} A_{27a}) (ap (c_{2Elist}_{2ESET}_{TO}_{LIST} A_{27a}) V0s)))))) \Rightarrow (30)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1f \in (A_{27a}^{ty_{2Enum}_{2Enum}}).(\forall V2n \in ty_{2Enum}_{2Enum}.((p (ap (c_{2Ebool}_{2EIN} A_{27a}) V0x) (ap (c_{2Elist}_{2ELIST}_{TO}_{SET} A_{27a}) (ap (ap (c_{2Elist}_{2EGENLIST} A_{27a}) V1f) V2n)))) \Leftrightarrow (\exists V3m \in ty_{2Enum}_{2Enum}.((p (ap (ap c_{2Eprim}_{rec}_{2E}_{3C} V3m) V2n)) \wedge (V0x = (ap V1f V3m)))))) \Rightarrow (31)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0f \in (A_{27a}^{ty_{2Enum}_{2Enum}}).(\forall V1n \in ty_{2Enum}_{2Enum}.((p (ap (c_{2Elist}_{2EALL}_{DISTINCT} A_{27a}) (ap (ap (c_{2Elist}_{2EGENLIST} A_{27a}) V0f) V1n)))) \Leftrightarrow (\forall V2m1 \in ty_{2Enum}_{2Enum}.(\forall V3m2 \in ty_{2Enum}_{2Enum}.(((p (ap (ap c_{2Eprim}_{rec}_{2E}_{3C} V3m2) V1n)) \wedge ((p (ap (ap c_{2Eprim}_{rec}_{2E}_{3C} V3m2) V1n)) \wedge ((ap V0f V2m1) = (ap V0f V3m2)))) \Rightarrow (V2m1 = V3m2)))))) \Rightarrow (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V0m) (ap c_2Epred_set_2Ecount V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (c_2Epred_set_2EFINITE ty_2Enum_2Enum) (ap c_2Epred_set_2Ecount V0n)))) \quad (34)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Erich_list_2ECOUNT_LIST V0n) = (ap (ap (c_2Elist_2EGENLIST ty_2Enum_2Enum) (c_2Ecombin_2E ty_2Enum_2Enum)) V0n))) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ & A_27a). (\forall V1l2 \in (ty_2Elist_2Elist A_27a). (((p (ap (c_2Elist_2EALL_DISTINCT \\ & A_27a) V0l1)) \wedge ((p (ap (c_2Elist_2EALL_DISTINCT A_27a) V1l2)) \wedge \\ & (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) (ap (\\ & c_2Elist_2ELIST_TO_SET A_27a) V0l1))) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN \\ & A_27a) V2x) (ap (c_2Elist_2ELIST_TO_SET A_27a) V1l2)))))) \Rightarrow \\ & (p (ap (ap (c_2Esorting_2Eperm A_27a) V0l1) V1l2)))))) \end{aligned} \quad (36)$$

Theorem 1

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap (c_2Esorting_2Eperm ty_2Enum_2Enum) (ap (c_2Elist_2ESET_TO_LIST ty_2Enum_2Enum) (ap c_2Epred_set_2Ecount V0n))) (ap c_2Erich_list_2ECOUNT_LIST V0n))))))$$