

thm_2Esorting_2EPERM_SINGLE_SWAP_APPEND
 (TMUCFvx-
 CtLS9f2TPKQsAw5nQBMLD8VWfQ8K)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (2)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V2t \in 2.V2t)))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (4)$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 8 We define $c_2Esorting_2E_PERM_SINGLE_SWAP$ to be $\lambda A_27a : \iota.\lambda V0l1 \in (ty_2Elist_2Elist$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(ap (ap (c_2Elist_2EAPPEND A_27a) (c_2Elist_2ENIL A_27a)) \\ & V0l) = V0l) \wedge (\forall V1l1 \in (ty_2Elist_2Elist A_27a).(\forall V2l2 \in \\ & (ty_2Elist_2Elist A_27a).(\forall V3h \in A_27a.((ap (ap (c_2Elist_2EAPPEND \\ & A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V3h) V1l1)) V2l2) = (ap (ap \\ & (c_2Elist_2ECONS A_27a) V3h) (ap (ap (c_2Elist_2EAPPEND A_27a) \\ & V1l1) V2l2)))))))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x1 \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1x2 \in (ty_2Elist_2Elist A_27a).(\forall V2x3 \in \\ & (ty_2Elist_2Elist A_27a).(p (ap (ap (c_2Esorting_2E_PERM_SINGLE_SWAP \\ & A_27a) (ap (ap (c_2Elist_2EAPPEND A_27a) (ap (ap (c_2Elist_2EAPPEND \\ & A_27a) V0x1) V1x2)) V2x3)) (ap (ap (c_2Elist_2EAPPEND A_27a) (ap \\ & (ap (c_2Elist_2EAPPEND A_27a) V0x1) V2x3)) V1x2)))))) \end{aligned} \tag{6}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x2 \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1x3 \in (ty_2Elist_2Elist A_27a).(p (ap (ap (c_2Esorting_2E_PERM_SINGLE_SWAP \\ & A_27a) (ap (ap (c_2Elist_2EAPPEND A_27a) V0x2) V1x3)) (ap (ap (c_2Elist_2EAPPEND \\ & A_27a) V1x3) V0x2)))) \end{aligned}$$