thm_2Esorting_2EPERM__SINGLE__SWAP__TC__CONS (TMT2dx5SNGuAKvh9ipNfXebCjZXHocBj9j4)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{2}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 9 *We define* c_2Esorting_2EPERM__SINGLE__SWAP *to be* $\lambda A\_27a:\iota.\lambda V0l1 \in (ty\_2Elist\_2Elist$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V1x)\ V2y))))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in A\_27a.(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V3x)\ V4y)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V4y)\ V5z))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V3x)\ V5z)))))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V1x)\ V2y)))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1P \in ((2^{A\_27a})^{A\_27a}).(((\forall V2x \in A\_27a.(\forall V3y \in A\_27a.((p\ (ap\ (ap\ V0R\ V2x)\ V3y)) \Rightarrow (p\ (ap\ (ap\ V1P\ V2x)\ V3y))))) \wedge (\forall V4x \in A\_27a.(\forall V5y \in A\_27a.(\forall V6z \in A\_27a.(((p\ (ap\ (ap\ V1P\ V4x)\ V5y)) \wedge (p\ (ap\ (ap\ V1P\ V5y)\ V6z))) \Rightarrow (p\ (ap\ (ap\ V1P\ V4x)\ V6z))))))) \Rightarrow (\forall V7u \in A\_27a.(\forall V8v \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V7u)\ V8v)) \Rightarrow (p\ (ap\ (ap\ V1P\ V7u)\ V8v)))))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0M \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1N \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a)\ V0M)\ V1N)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ V0M))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x)\ V1N))))))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1M \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2N \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a))\ V1M)\ V2N)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1M))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2N)))))))$$