

# thm\_2Esorting\_2EPERM\_STRONG\_IND (TMM6wRoY8vLsza4uZtL2217L3wkbCJzzb59)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (2)$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))^{A\_27a}) \quad (3)$$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EFILTER A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))^{(2^{A\_27a})}) \quad (4)$$

**Definition 8** We define  $c\_2\text{Esorting\_2EPERM}$  to be  $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist A\_27a).\lambda V1L2 \in$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0x \in A\_27a.(\forall V1y \in \\ & A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p V0t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in \\ & 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w))) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow \\ & ((p V1y) \wedge (p V3w)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0x \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1y \in (ty\_2Elist\_2Elist A\_27a).(\forall V2z \in \\ & (ty\_2Elist\_2Elist A\_27a).(((p (ap (ap (c\_2Esorting\_2EPERM A\_27a) \\ & V0x) V1y)) \wedge (p (ap (ap (c\_2Esorting\_2EPERM A\_27a) V1y) V2z))) \Rightarrow ( \\ & p (ap (ap (c\_2Esorting\_2EPERM A\_27a) V0x) V2z)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist A\_27a).(\forall V2x \in \\ & A\_27a.((p (ap (ap (c\_2Esorting\_2EPERM A\_27a) V0l1) V1l2)) \Rightarrow (p ( \\ & ap (ap (c\_2Esorting\_2EPERM A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) \\ & V2x) V0l1)) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2x) V1l2)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0L \in (ty\_2Elist\_2Elist \\ & A\_27a).(((p (ap (ap (c\_2Esorting\_2EPERM A\_27a) V0L) (c\_2Elist\_2ENIL \\ & A\_27a))) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL A\_27a))) \wedge ((p (ap (ap (c\_2Esorting\_2EPERM \\ & A\_27a) (c\_2Elist\_2ENIL A\_27a)) V0L)) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL A\_27a)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in ((2^{(ty\_2Elist\_2Elist\ A.27a)})(ty\_2Elist\_2Elist\ A.27a)), \\
& \quad (((p\ (ap\ (ap\ V0P\ (c.2Elist\_2ENIL\ A.27a))\ (c.2Elist\_2ENIL\ A.27a)))) \wedge \\
& \quad \quad ((\forall V1x \in A.27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ A.27a). \\
& \quad \quad (\forall V3l2 \in (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ V0P\ V2l1)\ V3l2)) \Rightarrow \\
& \quad \quad (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V1x)\ V2l1))\ (ap\ (ap \\
& \quad \quad (c.2Elist\_2ECONS\ A.27a)\ V1x)\ V3l2)))))) \wedge ((\forall V4x \in A.27a. \\
& \quad \quad (\forall V5y \in A.27a.(\forall V6l1 \in (ty\_2Elist\_2Elist\ A.27a). \\
& \quad \quad (\forall V7l2 \in (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ V0P\ V6l1)\ V7l2)) \Rightarrow \\
& \quad \quad (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V4x)\ (ap\ (ap\ (c.2Elist\_2ECONS \\
& \quad \quad A.27a)\ V5y)\ V6l1)))\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V5y)\ (ap\ (ap \\
& \quad \quad (c.2Elist\_2ECONS\ A.27a)\ V4x)\ V7l2)))))) \wedge ((\forall V8l1 \in (ty\_2Elist\_2Elist \\
& \quad \quad A.27a).(\forall V9l2 \in (ty\_2Elist\_2Elist\ A.27a).(\forall V10l3 \in \\
& \quad \quad (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ V0P\ V8l1)\ V9l2)) \wedge (p\ (ap\ ( \\
& \quad \quad ap\ V0P\ V9l2)\ V10l3))) \Rightarrow (p\ (ap\ (ap\ V0P\ V8l1)\ V10l3)))))) \Rightarrow (\forall V11l1 \in \\
& \quad \quad (ty\_2Elist\_2Elist\ A.27a).(\forall V12l2 \in (ty\_2Elist\_2Elist \\
& \quad \quad A.27a).((p\ (ap\ (ap\ (c.2Esorting\_2Eperm\ A.27a)\ V11l1)\ V12l2)) \Rightarrow \\
& \quad \quad (p\ (ap\ (ap\ V0P\ V11l1)\ V12l2))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& \quad A.27a.(\forall V2l1 \in (ty\_2Elist\_2Elist\ A.27a).(\forall V3l2 \in \\
& \quad (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ (c.2Esorting\_2Eperm\ A.27a) \\
& \quad (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V0x)\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a) \\
& \quad V1y)\ V2l1)))\ (ap\ (ap\ (c.2Elist\_2ECONS\ A.27a)\ V1y)\ (ap\ (ap\ (c.2Elist\_2ECONS \\
& \quad A.27a)\ V0x)\ V3l2)))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Esorting\_2Eperm\ A.27a)\ V2l1) \\
& \quad V3l2))))))
\end{aligned} \tag{14}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})(ty\_2Elist\_2Elist\ A\_27a)), \\
& \quad (((p\ (ap\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge \\
& \quad \quad ((\forall V1x \in A\_27a. (\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a). \\
& \quad \quad (\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM \\
& \quad \quad A\_27a)\ V2l1)\ V3l2)) \wedge (p\ (ap\ (ap\ V0P\ V2l1)\ V3l2))) \Rightarrow (p\ (ap\ (ap\ V0P\ (ap \\
& \quad \quad (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1x)\ V2l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad \quad A\_27a)\ V1x)\ V3l2)))))) \wedge ((\forall V4x \in A\_27a. (\forall V5y \in A\_27a. \\
& \quad \quad (\forall V6l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V7l2 \in (ty\_2Elist\_2Elist \\
& \quad \quad A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V6l1)\ V7l2)) \wedge ( \\
& \quad \quad p\ (ap\ (ap\ V0P\ V6l1)\ V7l2))) \Rightarrow (p\ (ap\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad \quad A\_27a)\ V4x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ V6l1)))\ (ap\ (ap \\
& \quad \quad (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x) \\
& \quad \quad V7l2)))))) \wedge ((\forall V8l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V9l2 \in \\
& \quad \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V10l3 \in (ty\_2Elist\_2Elist \\
& \quad \quad A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a)\ V8l1)\ V9l2)) \wedge ( \\
& \quad \quad (p\ (ap\ (ap\ V0P\ V8l1)\ V9l2)) \wedge ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a) \\
& \quad \quad V9l2)\ V10l3)) \wedge (p\ (ap\ (ap\ V0P\ V9l2)\ V10l3)))))) \Rightarrow (p\ (ap\ (ap\ V0P\ V8l1) \\
& \quad \quad V10l3)))))) \Rightarrow (\forall V11l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V12l2 \in \\
& \quad \quad (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Esorting\_2EPERM\ A\_27a) \\
& \quad \quad V11l1)\ V12l2)) \Rightarrow (p\ (ap\ (ap\ V0P\ V11l1)\ V12l2))))))
\end{aligned}$$