thm_2Esorting_2EPERM__TC (TMWgEGU-jtVezAkWGh8bSnXMqVgihUGQ43cj)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\text{c\_2Ebool\_2E}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{3}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{4}$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{5}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{6}$$

Definition 10 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$**.if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Definition 13 *We define* c_2Esorting_2EPERM__SINGLE__SWAP *to be* $\lambda A\_27a : \iota.\lambda V0l1 \in (ty\_2Elist\_2Elis$

Definition 14 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{9}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{10}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n))))) \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{19}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2)))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V0l1))\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1l2))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1L \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2M \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V0P)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1L)\ V2M)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V0P)\ V1L))\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ V0P)\ V2M)))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V1x)\ V2y))))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.(\forall V5z \in A\_27a.(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V3x)\ V4y)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V4y)\ V5z))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V3x)\ V5z))))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V1x)\ V2y)))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).\ ((p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ V0R)) \Rightarrow ((ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R) = V0R)))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).\ (\forall V1Q \in ((2^{A\_27a})^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27a.((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((p\ (ap\ (ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V1Q\ V4x)\ V5y))))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V2x)\ V3y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V1Q)\ V2x)\ V3y)))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0L \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0L)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL\ A\_27a))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0L)) \Leftrightarrow (V0L = (c\_2Elist\_2ENIL\ A\_27a))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1L \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2h \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V0t))\ V1L)) \Leftrightarrow (\exists V3M \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V4N \in (ty\_2Elist\_2Elist\ A\_27a).((V1L = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3M)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V4N))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0t)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V3M)\ V4N)))))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1L2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0L1)\ V1L2)) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V2x))\ V0L1)) = (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A\_27a)\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V2x))\ V1L2))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1x3 \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0x2)\ V1x3))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1x3)\ V0x2)))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a)\ V0l)\ V0l))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1M \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2N \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a))\ V1M)\ V2N)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V1M))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ V2N))))))) \tag{36}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\mathsf{c\_2Esorting\_2EPERM}\ A\_27a) = (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a)))$$