

thm_2Esorting_2Eperm_alt (TMXYettRcvXbZ9zJn2TUpbVy3rGieJtMT68)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (2)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Esorting_2Eperm$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist A_27a).\lambda V1L2 \in$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (6)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1l1 \in \\ & \quad (ty_2Elist_2Elist\ A.27a).(\forall V2l2 \in (ty_2Elist_2Elist\ A.27a). \\ & \quad ((ap\ (c_2Elist_2ELENGTH\ A.27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A.27a) \\ & \quad (ap\ (c_2Emin_2E_3D\ A.27a)\ V0x))\ V1l1)) = (ap\ (c_2Elist_2ELENGTH\ A.27a) \\ & \quad (ap\ (ap\ (c_2Elist_2EFILTER\ A.27a)\ (ap\ (c_2Emin_2E_3D\ A.27a) \\ & \quad V0x))\ V2l2))) \Rightarrow ((ap\ (ap\ (c_2Elist_2EFILTER\ A.27a)\ (ap\ (c_2Emin_2E_3D \\ & \quad A.27a)\ V0x))\ V1l1) = (ap\ (ap\ (c_2Elist_2EFILTER\ A.27a)\ (ap\ (c_2Emin_2E_3D \\ & \quad A.27a)\ V0x))\ V2l2)))))) \end{aligned} \quad (8)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0L1 \in (ty_2Elist_2Elist \\ & \quad A.27a).(\forall V1L2 \in (ty_2Elist_2Elist\ A.27a).((p\ (ap\ (ap\ (c_2Esorting_2EPERM \\ & \quad A.27a)\ V0L1)\ V1L2)) \Leftrightarrow (\forall V2x \in A.27a.((ap\ (c_2Elist_2ELENGTH \\ & \quad A.27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A.27a)\ (ap\ (c_2Emin_2E_3D\ A.27a) \\ & \quad V2x))\ V0L1)) = (ap\ (c_2Elist_2ELENGTH\ A.27a)\ (ap\ (ap\ (c_2Elist_2EFILTER \\ & \quad A.27a)\ (ap\ (c_2Emin_2E_3D\ A.27a)\ V2x))\ V1L2)))))) \end{aligned}$$