

thm_2Esorting_2Eperm_lifts_equalities
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SrmeA4KzYYhZY2LMLcxjrHJT4mM)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2Eappend : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2Eappend A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(ty_2Elist_2Elist A_27a)) \quad (2)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Esorting_2Eperm_single_swap$ to be $\lambda A_27a : \iota.\lambda V0t1 \in (ty_2Elist_2Elist A_27a)$

Definition 12 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (3)$$

Definition 13 We define $c_2Esorting_2EPERM$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist\ A_27a).\lambda V1L2$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (8)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (9)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (12)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (13)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap\ V0f\ V2x)))) \Leftrightarrow (p (ap\ V0f\ V1v)))))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1f \in (A_{.27b}^{A_{.27a}}).((\forall V2x \in A_{.27a}.(\forall V3y \in A_{.27a}.((p (ap (ap\ V0R\ V2x)\ V3y)) \Rightarrow ((ap\ V1f\ V2x) = (ap\ V1f\ V3y)))))) \Rightarrow (\forall V4x \in A_{.27a}.(\forall V5y \in A_{.27a}.((p (ap (ap (ap (c_{.2Erelation_2ETC}\ A_{.27a})\ V0R)\ V4x)\ V5y)) \Rightarrow ((ap\ V1f\ V4x) = (ap\ V1f\ V5y)))))))))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((c_{.2Esorting_2EPERM}\ A_{.27a}) = (ap (c_{.2Erelation_2ETC}\ (ty_{.2Elist_2Elist}\ A_{.27a})) (c_{.2Esorting_2EPERM_SINGLE_SWAP}\ A_{.27a}))) \quad (16)$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0f \in (A_{.27b}^{(ty_{.2Elist_2Elist}\ A_{.27a})}).((\forall V1x1 \in (ty_{.2Elist_2Elist}\ A_{.27a}).(\forall V2x2 \in (ty_{.2Elist_2Elist}\ A_{.27a}).(\forall V3x3 \in (ty_{.2Elist_2Elist}\ A_{.27a}).((ap\ V0f (ap (ap (c_{.2Elist_2EAPPEND}\ A_{.27a}) (ap (ap (c_{.2Elist_2EAPPEND}\ A_{.27a})\ V1x1)\ V2x2))\ V3x3)) = (ap\ V0f (ap (ap (c_{.2Elist_2EAPPEND}\ A_{.27a}) (ap (ap (c_{.2Elist_2EAPPEND}\ A_{.27a})\ V1x1)\ V3x3))\ V2x2)))))) \Rightarrow (\forall V4x \in (ty_{.2Elist_2Elist}\ A_{.27a}).(\forall V5y \in (ty_{.2Elist_2Elist}\ A_{.27a}).((p (ap (ap (c_{.2Esorting_2EPERM}\ A_{.27a})\ V4x)\ V5y)) \Rightarrow ((ap\ V0f\ V4x) = (ap\ V0f\ V5y)))))))$$