thm_2Esorting_2EPERM__lifts__monotonicities (TMPk6G3PgrxC3eseoLv4awGwXHnKrMKVXHB)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^{2}))$ $(\lambda V0x \in 2.V0x))$ $(\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)$ $(\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{2}$$

Definition 9 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2 \in$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)$ $(\lambda V2t \in$

Definition 11 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{5}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))\ V2l3))))) \tag{8}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a))) \tag{9}$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V0l1))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V1l1))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l)\ V2l2))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1l1)\ V2l2)))))) \land (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V4l1)\ V3l))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5l2)\ V3l))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V4l1)\ V5l2)))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)}).(\forall V1Q \in ((2^{A\_27b})^{A\_27b}).(((\forall V2x1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3x2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4x3 \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ V1Q\ (ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2x1)\ V3x2))\ V4x3)))\ (ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2x1)\ V4x3))\ V3x2))))))) \land (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27b)\ V1Q))) \Rightarrow (\forall V5x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6y \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V5x)\ V6y)) \Rightarrow (p\ (ap\ (ap\ V1Q\ (ap\ V0f\ V5x))\ (ap\ V0f\ V6y))))))))) \tag{12}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)}).((\forall V1x1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2x2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3x3 \in (ty\_2Elist\_2Elist\ A\_27a).(\exists V4x1\_27 \in (ty\_2Elist\_2Elist\ A\_27b).(\exists V5x2\_27 \in (ty\_2Elist\_2Elist\ A\_27b).(\exists V6x3\_27 \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1x1)\ V2x2))\ V3x3)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ V4x1\_27)\ V5x2\_27))\ V6x3\_27)) \land ((ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1x1)\ V3x3))\ V2x2)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ V4x1\_27)\ V6x3\_27))\ V5x2\_27)))))))) \Rightarrow (\forall V7x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V8y \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V7x)\ V8y)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27b)\ (ap\ V0f\ V7x))\ (ap\ V0f\ V8y))))))))$$