thm_2Esorting_2EPERM__lifts__transitive__relations (TMcjxoLJSPvSw1ziVVMyKqK2yxTibT9vh6k)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (c\_2Ebool\_2E$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 12 *We define* c_2Esorting_2EPERM__SINGLE__SWAP *to be* $\lambda A\_27a : \iota.\lambda V0l1 \in (ty\_2Elist\_2Elis$

Definition 13 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{3}$$

Definition 14 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (p\ V1Q))))) \tag{8}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{9}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{10}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{12}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v)\Rightarrow(p\ (ap\ V0f\ V2x))))\Leftrightarrow(p\ (ap\ V0f\ V1v))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1Q \in ((2^{A\_27b})^{A\_27b}). (\forall V2f \in (A\_27b^{A\_27a}).(((\forall V3x \in A\_27a.(\forall V4y \in A\_27a.((p\ (ap\ (ap\ V0R\ V3x)\ V4y))\Rightarrow(p\ (ap\ (ap\ V1Q\ (ap\ V2f\ V3x))\ (ap\ V2f\ V4y))))))\wedge(p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27b)\ V1Q)))\Rightarrow(\forall V5x \in A\_27a.(\forall V6y \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V5x)\ V6y))\Rightarrow(p\ (ap\ (ap\ V1Q\ (ap\ V2f\ V5x))\ (ap\ V2f\ V6y)))))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\mathsf{c\_2Esorting\_2EPERM}\ A\_27a) = (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Elist\_2Elist\ A\_27a))\ (\mathsf{c\_2Esorting\_2EPERM\_\_SINGLE\_\_SWAP}\ A\_27a))) \tag{16}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)}).(\forall V1Q \in ((2^{A\_27b})^{A\_27b}).(((\forall V2x1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V3x2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4x3 \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ V1Q\ (ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2x1)\ V3x2))\ V4x3)))\ (ap\ V0f\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2x1)\ V4x3))\ V3x2)))))))\wedge(p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27b)\ V1Q)))\Rightarrow(\forall V5x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6y \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V5x)\ V6y))\Rightarrow(p\ (ap\ (ap\ V1Q\ (ap\ V0f\ V5x))\ (ap\ V0f\ V6y)))))))))$$