

thm_2Esorting_2ESORTED_FILTER (TM- ctk7wh5c4YBn16iT93RK4RXXNZwRcPLFb)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \tag{1}$$

Let `c_2Elist_2EFILTER` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EFILTER A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \tag{2}$$

Definition 10 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let `c_2Elist_2ELIST_TO_SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \tag{3}$$

Definition 11 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 12 We define $c_ERelation_2Etransitive$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_Ebool_2EIN))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (4)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (5)$$

Let $c_2Esorting_2ESORTED : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Esorting_2ESORTED\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)^{(2^{A_27a})^{A_27a}})}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (10)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t) \Leftrightarrow (p\ V1x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ & V5y_27)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((ap\ (\\
& ap\ (c_2Elist_2EFILTER\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL \\
& A_27a))) \wedge (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in \\
& (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\
& V1P)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& (ty_2Elist_2Elist\ A_27a))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V1P)\ V3t))))\ (ap\ (ap \\
& (c_2Elist_2EFILTER\ A_27a)\ V1P)\ V3t))))))))) \\
& \hspace{15em} (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1L \in \\
& (ty_2Elist_2Elist\ A_27a).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER \\
& A_27a)\ V0P)\ V1L)))) \Leftrightarrow ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1L))))))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a.((p\ (ap\ (ap \\
& (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a) \\
& (c_2Elist_2ENIL\ A_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A_27a.(\forall V2h \in \\
& A_27a.(\forall V3t \in (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V3t))))))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})^{A_27a}})). \\
& (((\forall V1R \in ((2^{A_27a})^{A_27a}).(p\ (ap\ (ap\ V0P\ V1R)\ (c_2Elist_2ENIL \\
& A_27a)))) \wedge ((\forall V2R \in ((2^{A_27a})^{A_27a}).(\forall V3x \in A_27a. \\
& (p\ (ap\ (ap\ V0P\ V2R)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3x)\ (c_2Elist_2ENIL \\
& A_27a)))))) \wedge (\forall V4R \in ((2^{A_27a})^{A_27a}).(\forall V5x \in A_27a. \\
& (\forall V6y \in A_27a.(\forall V7rst \in (ty_2Elist_2Elist\ A_27a). \\
& ((p\ (ap\ (ap\ V0P\ V4R)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V6y)\ V7rst))) \Rightarrow \\
& (p\ (ap\ (ap\ V0P\ V4R)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V5x)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V6y)\ V7rst))))))))) \Rightarrow (\forall V8v \in ((2^{A_27a})^{A_27a}). \\
& (\forall V9v1 \in (ty_2Elist_2Elist\ A_27a).(p\ (ap\ (ap\ V0P\ V8v)\ V9v1)))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1L \in (ty_2Elist_2Elist\ A_27a). (\forall V2x \in A_27a. (\\
& (p\ (ap\ (c_2Erelation_2Etransitive\ A_27a)\ V0R)) \Rightarrow ((p\ (ap\ (ap\ (c_2Esorting_2ESORTED \\
& \quad A_27a)\ V0R)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2x)\ V1L))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Esorting_2ESORTED\ A_27a)\ V0R)\ V1L)) \wedge (\forall V3y \in A_27a. \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3y)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27a)\ V1L))) \Rightarrow (p\ (ap\ (ap\ V0R\ V2x)\ V3y))))))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (p\ (ap\ (ap\ (c_2Esorting_2ESORTED\ A_27a)\ V0R)\ (c_2Elist_2ENIL\ A_27a)))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& (\forall V1x \in A_27a. (p\ (ap\ (ap\ (c_2Esorting_2ESORTED\ A_27a)\ V0R) \\
& (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1x)\ (c_2Elist_2ENIL\ A_27a)))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1ls \in (ty_2Elist_2Elist\ A_27a). (\forall V2P \in (2^{A_27a}). \\
& (((p\ (ap\ (c_2Erelation_2Etransitive\ A_27a)\ V0R)) \wedge (p\ (ap\ (ap\ (c_2Esorting_2ESORTED \\
& \quad A_27a)\ V0R)\ V1ls))) \Rightarrow (p\ (ap\ (ap\ (c_2Esorting_2ESORTED\ A_27a)\ V0R) \\
& \quad (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ V2P)\ V1ls)))))))))
\end{aligned}$$