

thm_2Esptree_2EIN__domain
(TMazbj1xRaxsBwKvn8ESZKKeUJic9EpZ28A)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{6}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then}$ (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 15 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC (ap$

Definition 16 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enumeral_2EEVEN : \iota$ be given. Assume the following.

$$c_2Enumeral_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enumeral_2Eonecount : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eonecount \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enumeral_2Eexactlog : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexactlog \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 19 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Definition 20 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 23 We define $c_2Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $c_2Enumeral_2Eteexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eteexp_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (15)$$

Definition 24 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b))$

Definition 25 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 26 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 27 We define $c_2Enumeral_2Einternal_mult$ to be $c_2Earithmetic_2E_2A$.

Definition 28 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \quad (17)$$

Let $c_2Esptree_2EBS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esptree_2EBS A_27a \in (((ty_2Esptree_2Espt A_27a)^{(ty_2Esptree_2Espt A_27a)})^{(ty_2Esptree_2Espt A_27a)}) \quad (18)$$

Definition 29 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (19)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (20)$$

Definition 30 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(21)

Definition 31 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in$

Definition 32 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Esptree_2EBN : \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2EBN\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{(ty_2Esptree_2Espt\ A_27a)})$$
(22)

Definition 33 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Esptree_2ELS : \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2ELS\ A_27a \in ((ty_2Esptree_2Espt\ A_27a)^{A_27a})$$
(23)

Definition 34 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Let $c_2Esptree_2ELN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2ELN\ A_27a \in (ty_2Esptree_2Espt\ A_27a)$$
(24)

Let $c_2Esptree_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Edomain\ A_27a \in ((2^{ty_2Enum_2Enum})^{(ty_2Esptree_2Espt\ A_27a)})$$
(25)

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))$$
(26)

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m))$$
(27)

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
& \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1n) V0m)))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in \\
& \quad ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V0n))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V1n) V0m)))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\
& \quad V0m) c_2Enum_2E0) = V0m))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ (ap \\
& c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge \\
& (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0m) = V0m) \wedge \\
& (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap \\
& (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n)) \\
& V1n)) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2A \\
& V0m)\ V1n)))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2A \\
& V1n)\ V0m))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m) \\
& (ap\ (ap\ c_2Earithmetic_2E_2A\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2A \\
& (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
& c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\
& V2p) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2A V0m) \\
& \quad V1n) = (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee \\
& \quad \quad (V1n = V2p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2A V1n) \\
& \quad V0m) = (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee \\
& \quad \quad (V1n = V2p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) \\
& \quad \quad V1c)) V1c) = V0a))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
& \quad \quad c_2Enum_2E0) \vee (V1n = c_2Enum_2E0))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap c_2Earithmetic_2EODD V0n)) \Leftrightarrow \\
& \quad (\neg (p (ap c_2Earithmetic_2EEVEN V0n))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((p (ap c_2Earithmetic_2EEVEN \\
& \quad V0n)) \Rightarrow (\exists V1m \in ty_2Enum_2Enum. (V0n = (ap (ap c_2Earithmetic_2E_2A \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& \quad V1m)))) \wedge ((p (ap c_2Earithmetic_2EODD V0n)) \Rightarrow (\exists V2m \in ty_2Enum_2Enum. \\
& \quad (V0n = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V2m))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmic_2E_3C_3D V1n) V0m))))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D (ap (ap c_2Earithmic_2E_2B V0m) V1n)) (ap (ap c_2Earithmic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D V1n) V2p)))))) \quad (50)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (51)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmic_2E_2B (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))) V0n))) \quad (52)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D (ap (ap c_2Earithmic_2E_2A (ap c_2Enum_2ESUC V2p)) V0m)) (ap (ap c_2Earithmic_2E_2A (ap c_2Enum_2ESUC V2p)) V1n)))))) \quad (53)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1k \in ty_2Enum_2Enum.(\forall V2q \in ty_2Enum_2Enum.((\exists V3r \in ty_2Enum_2Enum.((V1k = (ap (ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2A V2q) V0n)) V3r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3r) V0n)))) \Rightarrow (ap (ap c_2Earithmic_2EDIV V1k) V0n) = V2q)))) \quad (54)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1k \in ty_2Enum_2Enum.(\forall V2r \in ty_2Enum_2Enum.((\exists V3q \in ty_2Enum_2Enum.((V1k = (ap (ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2A V3q) V0n)) V2r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V2r) V0n)))) \Rightarrow (ap (ap c_2Earithmic_2EMOD V1k) V0n) = V2r)))) \quad (55)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1r \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1r) V0n)) \Rightarrow (\forall V2q \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2E_2B (ap \\
& (ap c_2Earithmetic_2E_2A V2q) V0n)) V1r)) V0n) = V2q))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\
& V1k) V0n) = V1k))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD \\
& (ap (ap c_2Earithmetic_2E_2A V1k) V0n)) V0n) = c_2Enum_2E0))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (\forall V1j \in ty_2Enum_2Enum. (\forall V2k \in \\
& ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2EMOD V1j) V0n)) (ap (ap c_2Earithmetic_2EMOD \\
& V2k) V0n))) V0n) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2B \\
& V1j) V2k)) V0n))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD \\
& (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) V0n) = (ap (ap c_2Earithmetic_2EMOD \\
& V1k) V0n))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EDIV \\
& (ap (ap c_2Earithmetic_2E_2A V1q) V0n)) V0n) = V1q))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
& V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$True \quad (63)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee \neg(p \ V0t))) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (67)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (68)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge (p \ V1t2) \wedge (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (71)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (72)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (73)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (74)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (75)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (76)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (77)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (78)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (79)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))) \quad (80)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B)))) \quad (81)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (82)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (83)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))))) \Leftrightarrow (p\ (\\
& ap\ V0P\ V1a))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\
& (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{86}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \tag{87}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{90}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
& \tag{91}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Earithmic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
& \tag{92}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (93)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ c_2Earithmetic_2EVEN\ c_2Earithmetic_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c_2Earithmetic_2EVEN\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c_2Earithmetic_2EVEN\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))) \wedge \\
& \quad \quad ((\neg(p\ (ap\ c_2Earithmetic_2EODD\ c_2Earithmetic_2EZERO))) \wedge ((\neg(p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT2\ V0n)))) \wedge \\
& \quad \quad \quad (\neg(p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))))))))) \\
& \hspace{15em} (94)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount\ c_2Earithmetic_2EZERO\ V0x) = V0x)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& \quad (\forall V2x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ V2x) = (ap\ (ap\ c_2Enumeral_2Eonecount\ V1n)\ (ap\ c_2Enum_2ESUC\ V2x)))))) \wedge (\forall V3n \in ty_2Enum_2Enum. \\
& \quad (\forall V4x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount\ (ap\ c_2Earithmetic_2EBIT2\ V3n))\ V4x) = c_2Earithmetic_2EZERO)))))) \\
& \hspace{15em} (95)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Enumeral_2Exactlog\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge \\
& \quad ((\forall V0n \in ty_2Enum_2Enum. ((ap\ c_2Enumeral_2Exactlog\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = c_2Earithmetic_2EZERO)) \wedge (\forall V1n \in \\
& \quad ty_2Enum_2Enum. ((ap\ c_2Enumeral_2Exactlog\ (ap\ c_2Earithmetic_2EBIT2\ V1n)) = (ap\ (ap\ (c_2Ebool_2ELET\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \\
& \quad (\lambda V2x \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum) \\
& \quad \quad (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum) V2x) c_2Earithmetic_2EZERO)) \\
& \quad \quad \quad c_2Earithmetic_2EZERO) (ap\ c_2Earithmetic_2EBIT1\ V2x)))))) (ap\ c_2Enumeral_2Eonecount\ V1n) c_2Earithmetic_2EZERO)))))) \\
& \hspace{15em} (96)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \forall V2y \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2A c_2Earithmic_2EZERO) \\
& V0n) = c_2Earithmic_2EZERO) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& V0n) c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge ((ap \\
& (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) (ap \\
& c_2Earithmic_2EBIT1 V2y)) = (ap (ap c_2Enumeral_2Einternal_mult \\
& (ap c_2Earithmic_2EBIT1 V1x)) (ap c_2Earithmic_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) \\
& (ap c_2Earithmic_2EBIT2 V2y)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND \\
& ty_2Enum_2Enum) (ap c_2Earithmic_2EODD V3n)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V3n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V1x)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT1 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT1 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V4m \in \\
& ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V4m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V4m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V2y)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT1 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x)))) \wedge ((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT2 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V5m \in \\
& ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V6n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V5m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V5m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 \\
& V2y)))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap c_2Earithmic_2EODD \\
& V6n)) (ap (ap c_2Enumeral_2Eexp_help (ap c_2Earithmic_2EDIV2 \\
& V6n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 V1x)))) \\
& (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x))))))))) (97)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Enumeral_2Einternal_mult c_2Earithmetic_2EZERO) \\
V0n) = c_2Earithmetic_2EZERO) \wedge ((ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge ((ap \\
& (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\
V0n)) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enumeral_2EiDUB (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c_2Enumeral_2Einternal_mult (ap \\
& c_2Earithmetic_2EBIT2 V0n)) V1m) = (ap c_2Enumeral_2EiDUB (ap \\
& c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m)))))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap \\
(c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \tag{99}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) \\
V2x) (ap (ap (c_2Epred_set_2EUNION A_27a) V0s) V1t))) \Leftrightarrow ((p (ap \\
& (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \vee (p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V2x) V1t)))))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN A_27a) \\
V0x) (ap (c_2Epred_set_2EINSERT A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) V2s)))))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& ((p (ap (ap (c_2Ebool_2EIN A_27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE \\
& A_27a A_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap V2f V3x)) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V1s)))))))))
\end{aligned} \tag{102}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{103}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{104}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (105)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (106)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (107)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (108)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (109)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (110)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (111)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (112)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (113)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (114)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (115)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (116)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (117)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (((\text{ap } (c_2Esptree_2Edomain } A_27a) \\ & (c_2Esptree_2ELN } A_27a)) = (c_2Epred_set_2EEMPTY } ty_2Enum_2Enum)) \wedge \\ & ((\forall V0v0 \in A_27a. ((\text{ap } (c_2Esptree_2Edomain } A_27a) (\text{ap } (c_2Esptree_2ELS \\ & A_27a) V0v0)) = (\text{ap } (\text{ap } (c_2Epred_set_2EINSERT } ty_2Enum_2Enum) \\ & c_2Enum_2E0) (c_2Epred_set_2EEMPTY } ty_2Enum_2Enum)))) \wedge ((\\ & \forall V1t1 \in (ty_2Esptree_2Espt } A_27a). (\forall V2t2 \in (ty_2Esptree_2Espt \\ & A_27a). ((\text{ap } (c_2Esptree_2Edomain } A_27a) (\text{ap } (\text{ap } (c_2Esptree_2EBN \\ & A_27a) V1t1) V2t2)) = (\text{ap } (\text{ap } (c_2Epred_set_2EUNION } ty_2Enum_2Enum) \\ & (\text{ap } (\text{ap } (c_2Epred_set_2EIMAGE } ty_2Enum_2Enum } ty_2Enum_2Enum) \\ & (\lambda V3n \in ty_2Enum_2Enum. (\text{ap } (\text{ap } c_2Earithmetic_2E_2B } (\text{ap } (\text{ap } \\ & c_2Earithmetic_2E_2A } (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } c_2Earithmetic_2EBIT2 \\ & c_2Earithmetic_2EZERO)))) V3n)) (\text{ap } c_2Earithmetic_2ENUMERAL \\ & (\text{ap } c_2Earithmetic_2EBIT2 } c_2Earithmetic_2EZERO)))))) (\text{ap } (c_2Esptree_2Edomain \\ & A_27a) V1t1))) (\text{ap } (\text{ap } (c_2Epred_set_2EIMAGE } ty_2Enum_2Enum \\ & ty_2Enum_2Enum) (\lambda V4n \in ty_2Enum_2Enum. (\text{ap } (\text{ap } c_2Earithmetic_2E_2B \\ & (\text{ap } (\text{ap } c_2Earithmetic_2E_2A } (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } \\ & c_2Earithmetic_2EBIT2 } c_2Earithmetic_2EZERO)))) V4n)) (\text{ap } c_2Earithmetic_2ENUMERAL \\ & (\text{ap } c_2Earithmetic_2EBIT1 } c_2Earithmetic_2EZERO)))))) (\text{ap } (c_2Esptree_2Edomain \\ & A_27a) V2t2)))))) \wedge ((\forall V5t1 \in (ty_2Esptree_2Espt } A_27a). \\ & (\forall V6v1 \in A_27a. (\forall V7t2 \in (ty_2Esptree_2Espt } A_27a). \\ & ((\text{ap } (c_2Esptree_2Edomain } A_27a) (\text{ap } (\text{ap } (\text{ap } (c_2Esptree_2EBS \\ & A_27a) V5t1) V6v1) V7t2)) = (\text{ap } (\text{ap } (c_2Epred_set_2EUNION } ty_2Enum_2Enum) \\ & (\text{ap } (\text{ap } (c_2Epred_set_2EUNION } ty_2Enum_2Enum) (\text{ap } (\text{ap } (c_2Epred_set_2EINSERT \\ & ty_2Enum_2Enum) c_2Enum_2E0) (c_2Epred_set_2EEMPTY } ty_2Enum_2Enum)))) \\ & (\text{ap } (\text{ap } (c_2Epred_set_2EIMAGE } ty_2Enum_2Enum } ty_2Enum_2Enum) \\ & (\lambda V8n \in ty_2Enum_2Enum. (\text{ap } (\text{ap } c_2Earithmetic_2E_2B } (\text{ap } (\text{ap } \\ & c_2Earithmetic_2E_2A } (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } c_2Earithmetic_2EBIT2 \\ & c_2Earithmetic_2EZERO)))) V8n)) (\text{ap } c_2Earithmetic_2ENUMERAL \\ & (\text{ap } c_2Earithmetic_2EBIT2 } c_2Earithmetic_2EZERO)))))) (\text{ap } (c_2Esptree_2Edomain \\ & A_27a) V5t1))) (\text{ap } (\text{ap } (c_2Epred_set_2EIMAGE } ty_2Enum_2Enum \\ & ty_2Enum_2Enum) (\lambda V9n \in ty_2Enum_2Enum. (\text{ap } (\text{ap } c_2Earithmetic_2E_2B \\ & (\text{ap } (\text{ap } c_2Earithmetic_2E_2A } (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } \\ & c_2Earithmetic_2EBIT2 } c_2Earithmetic_2EZERO)))) V9n)) (\text{ap } c_2Earithmetic_2ENUMERAL \\ & (\text{ap } c_2Earithmetic_2EBIT1 } c_2Earithmetic_2EZERO)))))) (\text{ap } (c_2Esptree_2Edomain \\ & A_27a) V7t2))))))))) \end{aligned} \quad (118)$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0n \in ty_2Enum_2Enum. (\forall V1x \in A.27a. (\forall V2t1 \in \\
& \quad (ty_2Esptree_2Espt\ A.27a). (\forall V3t2 \in (ty_2Esptree_2Espt \\
A.27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0n)\ (ap\ (c_2Esptree_2Edomain \\
& \quad A.27b)\ (c_2Esptree_2ELN\ A.27b)))) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Enum_2Enum)\ V0n)\ (ap\ (c_2Esptree_2Edomain\ A.27a)\ (ap\ (c_2Esptree_2ELS \\
& \quad A.27a)\ V1x)))) \Leftrightarrow (V0n = c_2Enum_2E0)) \wedge (((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Enum_2Enum)\ V0n)\ (ap\ (c_2Esptree_2Edomain\ A.27a)\ (ap\ (ap\ (\\
& \quad c_2Esptree_2EBN\ A.27a)\ V2t1)\ V3t2)))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \wedge \\
& \quad (p\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ c_2Earithmetic_2EEVEN\ V0n)) \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ (ap\ (ap\ c_2Earithmetic_2EDIV \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) (ap\ (c_2Esptree_2Edomain \\
A.27a)\ V2t1))) (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ (ap\ (ap\ c_2Earithmetic_2EDIV \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) (ap\ (c_2Esptree_2Edomain \\
& \quad A.27a)\ V3t2)))))) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum) \\
& \quad V0n)\ (ap\ (c_2Esptree_2Edomain\ A.27a)\ (ap\ (ap\ (ap\ (c_2Esptree_2EBS \\
& \quad A.27a)\ V2t1)\ V1x)\ V3t2)))) \Leftrightarrow ((V0n = c_2Enum_2E0) \vee (p\ (ap\ (ap\ (ap\ (\\
& \quad c_2Ebool_2ECOND\ 2)\ (ap\ c_2Earithmetic_2EEVEN\ V0n)) (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad ty_2Enum_2Enum)\ (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ (ap\ c_2Earithmetic_2E_2D \\
& \quad V0n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))))) (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& \quad c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) (ap\ (c_2Esptree_2Edomain \\
A.27a)\ V2t1))) (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ (ap\ (ap\ c_2Earithmetic_2EDIV \\
& \quad (ap\ (ap\ c_2Earithmetic_2E_2D\ V0n)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) (ap\ (c_2Esptree_2Edomain \\
& \quad A.27a)\ V3t2)))))))))
\end{aligned}$$