

thm_2Esptree_2Edelete__compute
(TMSBDWG11eZmpBz5eJdhq6A9XGctdqbfGjk)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 6 We define $c_2Ebool_2E_2E_EF$ to be $(ap (c_2Ebool_2E_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E_EF$

Definition 8 We define $c_2Ebool_2E_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 10 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40 P))))$

Definition 12 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 13 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 14 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 16 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Definition 17 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2ECOND$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{7}$$

Definition 18 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{8}$$

Definition 19 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 20 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{9}$$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \tag{10}$$

Let $c_2Esptree_2Emk_BS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Emk_BS\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{A_27a})^{(ty_2Esptree_2Espt\ A_27a)} \quad (11)$$

Let $c_2Esptree_2EBS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2EBS\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{A_27a})^{(ty_2Esptree_2Espt\ A_27a)} \quad (12)$$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT2\ V0n))$.

Definition 22 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 23 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1\ V0n))$.

Definition 24 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Esptree_2Emk_BN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Emk_BN\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{A_27a})^{(ty_2Esptree_2Espt\ A_27a)} \quad (15)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Esptree_2EBN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2EBN\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{A_27a})^{(ty_2Esptree_2Espt\ A_27a)} \quad (17)$$

Let $c_2Esptree_2ELS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2ELS\ A_27a \in ((ty_2Esptree_2Espt\ A_27a)^{A_27a}) \quad (18)$$

Let $c_2Esptree_2ELN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2ELN\ A_27a \in (ty_2Esptree_2Espt\ A_27a) \quad (19)$$

Let $c_2Esptree_2Edelete : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Esptree_2Edelete\ A.27a \in (((ty_2Esptree_2Espt\ A.27a)^{(ty_2Esptree_2Espt\ A.27a)})^{ty_2Enum_2Enum}) \quad (20)$$

Assume the following.

$$(((p\ (ap\ c_2Earithmetic_2EEVEN\ c_2Enum_2E0)) \Leftrightarrow True) \wedge (\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ c_2Earithmetic_2EEVEN\ (ap\ c_2Enum_2ESUC\ V0n))) \Leftrightarrow (\neg(p\ (ap\ c_2Earithmetic_2EEVEN\ V0n)))))) \quad (21)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V0m)))))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2E_2D\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2D\ V0m)\ c_2Enum_2E0) = V0m))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0n)) (ap c_2Enum_2ESUC \\
& V1m)) = (ap (ap c_2Earithmetic_2E_2D V0n) V1m))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\
& V1n) V0m))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V2p) \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A \\
& V2p) V1n))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V0m)))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ V0n))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V2p)))) \Leftrightarrow ((V0m = c_2Enum_2E0) \vee (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V2p))))))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V2p)))) \Leftrightarrow ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V0m)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1n)\ V2p)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\forall V2z \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ V2z)) \Rightarrow (((ap\ (ap\ c_2Earithmetic_2EDIV\ V1y)\ V2z) = V0x) \Leftrightarrow ((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0x)\ V2z))\ V1y)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1y)\ (ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Enum_2ESUC\ V0x))\ V2z)))))))))) \quad (37)$$

Assume the following.

$$True \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{55}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
& \tag{56}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Earithmic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
& \tag{57}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap c_2Arithmetic_2EVEN c_2Arithmetic_2EZERO)) \wedge \\
& \quad ((p (ap c_2Arithmetic_2EVEN (ap c_2Arithmetic_2EBIT2 V0n))) \wedge \\
& \quad ((\neg (p (ap c_2Arithmetic_2EVEN (ap c_2Arithmetic_2EBIT1 V0n)))) \wedge \\
& \quad \quad ((\neg (p (ap c_2Arithmetic_2EODD c_2Arithmetic_2EZERO))) \wedge ((\\
& \quad \quad \neg (p (ap c_2Arithmetic_2EODD (ap c_2Arithmetic_2EBIT2 V0n)))) \wedge \\
& \quad (p (ap c_2Arithmetic_2EODD (ap c_2Arithmetic_2EBIT1 V0n)))))))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0k \in ty_2Enum_2Enum. \\
& ((ap\ (ap\ (c_2Esptree_2Edelete\ A.27a)\ V0k)\ (c_2Esptree_2ELN\ A.27a)) = \\
& (c_2Esptree_2ELN\ A.27a))) \wedge ((\forall V1k \in ty_2Enum_2Enum. (\forall V2a \in \\
& A.27a. ((ap\ (ap\ (c_2Esptree_2Edelete\ A.27a)\ V1k)\ (ap\ (c_2Esptree_2ELS \\
& A.27a)\ V2a)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esptree_2Espt \\
& A.27a))\ (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V1k)\ c_2Enum_2E0)) \\
& (c_2Esptree_2ELN\ A.27a))\ (ap\ (c_2Esptree_2ELS\ A.27a)\ V2a)))))) \wedge \\
& ((\forall V3k \in ty_2Enum_2Enum. (\forall V4t1 \in (ty_2Esptree_2Espt \\
& A.27a). (\forall V5t2 \in (ty_2Esptree_2Espt\ A.27a). ((ap\ (ap\ (c_2Esptree_2Edelete \\
& A.27a)\ V3k)\ (ap\ (ap\ (c_2Esptree_2EBN\ A.27a)\ V4t1)\ V5t2)) = (ap\ (ap \\
& (ap\ (c_2Ebool_2ECOND\ (ty_2Esptree_2Espt\ A.27a))\ (ap\ (ap\ (c_2Emin_2E_3D \\
& ty_2Enum_2Enum)\ V3k)\ c_2Enum_2E0))\ (ap\ (ap\ (c_2Esptree_2EBN\ A.27a) \\
& V4t1)\ V5t2))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esptree_2Espt\ A.27a)) \\
& (ap\ c_2Earithmetic_2EEVEN\ V3k))\ (ap\ (ap\ (c_2Esptree_2Emk_BN \\
& A.27a)\ (ap\ (ap\ (c_2Esptree_2Edelete\ A.27a)\ (ap\ (ap\ c_2Earithmetic_2EDIV \\
& (ap\ (ap\ c_2Earithmetic_2E_2D\ V3k)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))\ V4t1)) \\
& V5t2))\ (ap\ (ap\ (c_2Esptree_2Emk_BN\ A.27a)\ V4t1)\ (ap\ (ap\ (c_2Esptree_2Edelete \\
& A.27a)\ (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ (ap\ c_2Earithmetic_2E_2D \\
& V3k)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))\ V5t2)))))) \wedge \\
& ((\forall V6k \in ty_2Enum_2Enum. (\forall V7t1 \in (ty_2Esptree_2Espt \\
& A.27a). (\forall V8a \in A.27a. (\forall V9t2 \in (ty_2Esptree_2Espt \\
& A.27a). ((ap\ (ap\ (c_2Esptree_2Edelete\ A.27a)\ V6k)\ (ap\ (ap\ (ap\ (c_2Esptree_2EBS \\
& A.27a)\ V7t1)\ V8a)\ V9t2)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esptree_2Espt \\
& A.27a))\ (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V6k)\ c_2Enum_2E0)) \\
& (ap\ (ap\ (c_2Esptree_2EBN\ A.27a)\ V7t1)\ V9t2))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& (ty_2Esptree_2Espt\ A.27a))\ (ap\ c_2Earithmetic_2EEVEN\ V6k))\ (\\
& ap\ (ap\ (ap\ (c_2Esptree_2Emk_BS\ A.27a)\ (ap\ (ap\ (c_2Esptree_2Edelete \\
& A.27a)\ (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ (ap\ c_2Earithmetic_2E_2D \\
& V6k)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))\ V7t1))\ V8a) \\
& V9t2))\ (ap\ (ap\ (ap\ (c_2Esptree_2Emk_BS\ A.27a)\ V7t1)\ V8a)\ (ap\ (ap \\
& (c_2Esptree_2Edelete\ A.27a)\ (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap \\
& (ap\ c_2Earithmetic_2E_2D\ V6k)\ (ap\ c_2Earithmetic_2ENUMERAL\ (\\
& ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))\ V9t2))))))))) \\
& (59)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\
& \quad \forall V1t \in (ty_2Esptree_2Espt A_{.27a}). (\forall V2a \in A_{.27a}. (\\
& \quad \quad \forall V3t1 \in (ty_2Esptree_2Espt A_{.27a}). (\forall V4t2 \in (ty_2Esptree_2Espt \\
& \quad \quad \quad A_{.27a}). (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2ENUMERAL \\
& \quad \quad \quad \quad V0n)) V1t) = (ap (ap (c_2Esptree_2Edelete A_{.27a}) V0n) V1t)) \wedge (((\\
& \quad \quad \quad \quad \quad ap (ap (c_2Esptree_2Edelete A_{.27a}) c_2Enum_2E0) (c_2Esptree_2ELN \\
& \quad \quad \quad \quad \quad A_{.27a})) = (c_2Esptree_2ELN A_{.27a})) \wedge (((ap (ap (c_2Esptree_2Edelete \\
& \quad \quad \quad \quad \quad A_{.27a}) c_2Enum_2E0) (ap (c_2Esptree_2ELS A_{.27a}) V2a)) = (c_2Esptree_2ELN \\
& \quad \quad \quad \quad \quad A_{.27a})) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) c_2Enum_2E0) (\\
& \quad \quad \quad \quad \quad \quad ap (ap (c_2Esptree_2EBN A_{.27a}) V3t1) V4t2)) = (ap (ap (c_2Esptree_2EBN \\
& \quad \quad \quad \quad \quad \quad A_{.27a}) V3t1) V4t2)) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) c_2Enum_2E0) \\
& \quad \quad \quad \quad \quad \quad (ap (ap (ap (c_2Esptree_2EBS A_{.27a}) V3t1) V2a) V4t2)) = (ap (ap (c_2Esptree_2EBN \\
& \quad \quad \quad \quad \quad \quad A_{.27a}) V3t1) V4t2)) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) c_2Earithmetic_2EZERO) \\
& \quad \quad \quad \quad \quad \quad \quad (c_2Esptree_2ELN A_{.27a})) = (c_2Esptree_2ELN A_{.27a})) \wedge (((ap (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad (c_2Esptree_2Edelete A_{.27a}) c_2Earithmetic_2EZERO) (ap (c_2Esptree_2ELS \\
& \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V2a)) = (c_2Esptree_2ELN A_{.27a})) \wedge (((ap (ap (c_2Esptree_2Edelete \\
& \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) c_2Earithmetic_2EZERO) (ap (ap (c_2Esptree_2EBN A_{.27a}) \\
& \quad \quad \quad \quad \quad \quad \quad \quad V3t1) V4t2)) = (ap (ap (c_2Esptree_2EBN A_{.27a}) V3t1) V4t2)) \wedge (((\\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad ap (ap (c_2Esptree_2Edelete A_{.27a}) c_2Earithmetic_2EZERO) (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad (ap (ap (c_2Esptree_2EBS A_{.27a}) V3t1) V2a) V4t2)) = (ap (ap (c_2Esptree_2EBN \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V3t1) V4t2)) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad c_2Earithmetic_2EBIT1 V0n)) (c_2Esptree_2ELN A_{.27a})) = (c_2Esptree_2ELN \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a})) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (c_2Esptree_2ELS A_{.27a}) V2a)) = (ap (c_2Esptree_2ELS \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V2a)) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (ap (c_2Esptree_2EBN A_{.27a}) V3t1) V4t2)) = (ap (ap (c_2Esptree_2Emk_BN \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V3t1) (ap (ap (c_2Esptree_2Edelete A_{.27a}) V0n) V4t2))) \wedge \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (ap (ap (c_2Esptree_2EBS A_{.27a}) V3t1) V2a) V4t2)) = (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (ap (ap (c_2Esptree_2Emk_BS A_{.27a}) V3t1) V2a) (ap (ap (c_2Esptree_2Edelete \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V0n) V4t2))) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad c_2Earithmetic_2EBIT2 V0n)) (c_2Esptree_2ELN A_{.27a})) = (c_2Esptree_2ELN \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a})) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT2 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (c_2Esptree_2ELS A_{.27a}) V2a)) = (ap (c_2Esptree_2ELS \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V2a)) \wedge (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT2 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (ap (c_2Esptree_2EBN A_{.27a}) V3t1) V4t2)) = (ap (ap (c_2Esptree_2Emk_BN \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) (ap (ap (c_2Esptree_2Edelete A_{.27a}) V0n) V3t1)) V4t2)) \wedge \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (((ap (ap (c_2Esptree_2Edelete A_{.27a}) (ap c_2Earithmetic_2EBIT2 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad V0n)) (ap (ap (ap (c_2Esptree_2EBS A_{.27a}) V3t1) V2a) V4t2)) = (ap \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (ap (ap (c_2Esptree_2Emk_BS A_{.27a}) (ap (ap (c_2Esptree_2Edelete \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad A_{.27a}) V0n) V3t1)) V2a) V4t2)))))))))
\end{aligned}$$