

thm_2Esptree_2Edomain__map1 (TMHTxkBbX6moy69QoxVSzQiaN5Xbf4PcgdH)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 6 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{4}$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \tag{5}$$

Definition 7 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_A_27a))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Esptree_2Espt\ A0) \quad (7)$$

Let $c_2Esptree_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Esptree_2Edomain\ A_27a \in ((2^{ty_2Enum_2Enum})^{(ty_2Esptree_2Espt\ A_27a)}) \quad (8)$$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Esptree_2Emapi0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esptree_2Emapi0\ A_27a\ A_27b \in (((((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27b)})^{ty_2Enum_2Enum})^{(A_27a^{A_27b})})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 12 We define $c_2Esptree_2Emapi$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum})$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Esptree_2Elookup\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in (ty_2Eoption_2Eoption A_27a).(\forall V2y \in A_27b.(((ap (ap (c_2Eoption_2EOPTION_MAP A_27a A_27b) V0f) V1x) = (ap (c_2Eoption_2ESOME A_27b) V2y)) \Leftrightarrow (\exists V3z \in A_27a.((V1x = (ap (c_2Eoption_2ESOME A_27a) V3z)) \wedge (V2y = (ap V0f V3z)))))))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt A_27a).(\forall V1k \in ty_2Enum_2Enum.((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V1k) (ap (c_2Esptree_2Edomain A_27a) V0t)) \Leftrightarrow (\exists V2v \in A_27a.((ap (ap (c_2Esptree_2Elookup A_27a) V1k) V0t) = (ap (c_2Eoption_2ESOME A_27a) V2v)))))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0k \in ty_2Enum_2Enum. (\forall V1f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum}). \\
& \quad (\forall V2pt \in (ty_2Esptree_2Espt\ A_27b). ((ap\ (ap\ (c_2Esptree_2Elookup \\
& \quad A_27a)\ V0k)\ (ap\ (ap\ (c_2Esptree_2Emapi\ A_27a\ A_27b)\ V1f)\ V2pt)) = \\
& \quad (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27a)\ (ap\ V1f\ V0k))\ (ap \\
& \quad \quad (ap\ (c_2Esptree_2Elookup\ A_27b)\ V0k)\ V2pt))))))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum}). (\forall V1pt \in (ty_2Esptree_2Espt \\
& \quad A_27b). ((ap\ (c_2Esptree_2Edomain\ A_27a)\ (ap\ (ap\ (c_2Esptree_2Emapi \\
& \quad A_27a\ A_27b)\ V0f)\ V1pt)) = (ap\ (c_2Esptree_2Edomain\ A_27b)\ V1pt)))
\end{aligned}$$