

# thm\_2Esptree\_2EfromAList\_def (TMVQe- seX4mVe18UHUKcEUUmr8PJpH6HvPGU)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2Elist\_2Elist\_size : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2Elist\_size\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}\ (ty\_2Enum\_2Enum^{A\_27a})) \quad (4)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2R \in 2.V2R)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (9)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num)$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2)))$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$

**Definition 17** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$



**Definition 27** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair$ .  
Let  $c\_2Esptree\_2ELN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2ELN\ A\_27a \in (ty\_2Esptree\_2Espt\ A\_27a) \quad (21)$$

**Definition 28** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 29** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 30** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A\_27a)$

Let  $c\_2Elist\_2Elist\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (22)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (23)$$

**Definition 31** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (24)$$

**Definition 32** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (c\_2Ebool\_2E\_2I$

**Definition 33** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in A\_27a.$

**Definition 34** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in A\_27a.$

**Definition 35** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in A\_27a.$

**Definition 36** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in A\_27a.$

**Definition 37** We define  $c\_2Esptree\_2EfromAList$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Erelation\_2EWFREC\ (ty\_2Elist\_2Elist\ A\_27a)$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
& ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))))) \\
& \tag{25}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V0m)))) \\
& \tag{26}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V0m)))) \\
& \tag{27}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
& \tag{28}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& c\_2Enum\_2E0) V0n))) \\
& \tag{29}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1n) V0m)))))) \\
& \tag{30}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{33}$$

Assume the following.

$$True \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{35}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Ecombin\_2El\ A\_27a)\ V0x) = V0x)) \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b.(\forall V1f \in ((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). \\ & ((ap\ (ap\ (c\_2Elist\_2Elist\_CASE\ A\_27a\ A\_27b)\ (c\_2Elist\_2ENIL \\ & A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A\_27a.(\forall V3a1 \in \\ & (ty\_2Elist\_2Elist\ A\_27a).(\forall V4v \in A\_27b.(\forall V5f \in ( \\ & (A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Elist\_2Elist\_CASE \\ & A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\ & (ap\ (ap\ V5f\ V2a0)\ V3a1)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & ((ap\ (ap\ (c\_2Elist\_2Elist\_size\ A\_27a)\ V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) = \\ & c\_2Enum\_2E0)) \wedge (\forall V1f \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2a0 \in \\ & A\_27a.(\forall V3a1 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2Elist\_size \\ & A\_27a)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0)\ V3a1)) = (ap\ (ap \\ & c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ V1f \\ & V2a0))\ (ap\ (ap\ (c\_2Elist\_2Elist\_size\ A\_27a)\ V1f)\ V3a1)))))) \end{aligned} \quad (42)$$





Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D c\_2Earithmic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\
& nonempty A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& A\_27a. (\forall V2y \in A\_27b. ((ap (ap (c\_2Epair\_2EUNCURRY A\_27a \\
& A\_27b A\_27c) V0f) (ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27b) V1x) V2y))) = \\
& (ap (ap V0f V1x) V2y))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\
& nonempty A\_27c \Rightarrow (\forall V0x \in A\_27b. (\forall V1y \in A\_27c. (\forall V2f \in \\
& ((A\_27a^{A\_27c})^{A\_27b}). ((ap (ap (c\_2Epair\_2Epair\_CASE A\_27a A\_27b \\
& A\_27c) (ap (ap (c\_2Epair\_2E\_2C A\_27b A\_27c) V0x) V1y)) V2f) = (ap \\
& (ap V2f V0x) V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(p (ap (c\_2Erelation\_2EWF ty\_2Enum\_2Enum) c\_2Eprim\_rec\_2E\_3C)) \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0R \in ((2^{A\_27b})^{A\_27a}). (\forall V1f \in (A\_27b^{A\_27a}). (( \\
& p (ap (c\_2Erelation\_2EWF A\_27b) V0R)) \Rightarrow (p (ap (c\_2Erelation\_2EWF \\
& A\_27a) (ap (ap (c\_2Erelation\_2Einv\_image A\_27a A\_27b) V0R) V1f))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}).(\forall V2y \in \\
& \quad A\_27a.(\forall V3z \in A\_27a.((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap \\
& \quad (ap\ (c\_2Erelation\_2ERESTRICT\ A\_27a\ A\_27b)\ V0f)\ V1R)\ V3z)\ V2y) = \\
& \quad (ap\ V0f\ V2y))))))))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (c\_2Erelation\_2EWFREC \\
& \quad A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ V1R)) \Rightarrow \\
& \quad (\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c\_2Erelation\_2ERESTRICT \\
& \quad A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x))))))))) \\
& \hspace{15em} (50)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Esptree\_2EfromAList \\
& \quad A\_27a)\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a))) = \\
& \quad (c\_2Esptree\_2ELN\ A\_27a)) \wedge (\forall V0y \in A\_27a.(\forall V1xs \in \\
& \quad (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)). \\
& \quad (\forall V2x \in ty\_2Enum\_2Enum.((ap\ (c\_2Esptree\_2EfromAList\ A\_27a) \\
& \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)) \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ A\_27a)\ V2x)\ V0y))\ V1xs)) = \\
& \quad (ap\ (ap\ (ap\ (c\_2Esptree\_2Einsert\ A\_27a)\ V2x)\ V0y)\ (ap\ (c\_2Esptree\_2EfromAList \\
& \quad A\_27a)\ V1xs)))))))))
\end{aligned}$$