

thm_2Esptree_2Einsert_shadow
(TMH6YGeKkuzdwd56MrL2b85UdGq9QywwDKG)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_COND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (2)$$

Let $c_2Esptree_2Einsert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Esptree_2Einsert A_27a \in (((ty_2Esptree_2Espt A_27a)^{(ty_2Esptree_2Espt A_27a)} A_27a)^{ty_2Enum_2Enum}) \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x1 \in ty_2Enum_2Enum. \\ (\forall V1x2 \in ty_2Enum_2Enum. (\forall V2v1 \in A_27a. (\forall V3v2 \in \\ A_27a. (\forall V4t \in (ty_2Esptree_2Espt\ A_27a). ((ap\ (ap\ (ap\ (c_2Esptree_2Einsert \\ A_27a)\ V0x1)\ V2v1)\ (ap\ (ap\ (ap\ (c_2Esptree_2Einsert\ A_27a)\ V1x2) \\ V3v2)\ V4t)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esptree_2Espt\ A_27a)) \\ (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V0x1)\ V1x2))\ (ap\ (ap\ (ap \\ (c_2Esptree_2Einsert\ A_27a)\ V0x1)\ V2v1)\ V4t))\ (ap\ (ap\ (ap\ (c_2Esptree_2Einsert \\ A_27a)\ V1x2)\ V3v2)\ (ap\ (ap\ (ap\ (c_2Esptree_2Einsert\ A_27a)\ V0x1) \\ V2v1)\ V4t)))))))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt \\ & A_27a). (\forall V1a \in ty_2Enum_2Enum. (\forall V2b \in A_27a. (\forall V3c \in \\ & A_27a. ((ap (ap (ap (c_2Esptree_2Einsert A_27a) V1a) V2b) (ap (ap \\ & (ap (c_2Esptree_2Einsert A_27a) V1a) V3c) V0t)) = (ap (ap (ap (c_2Esptree_2Einsert \\ & A_27a) V1a) V2b) V0t)))))) \end{aligned}$$