

thm_2Esptree_2Elookup__NONE__domain (TMan5pien1fTZdGuFQyTMuz6YB3XLdzy1Kx)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_2E21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define `c_2Ebool_2E_2EF` to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define `c_2Ebool_2E_5C_2E_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Eone_2Eone` to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 10 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E21 2))$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let `c_2Esum_2EABS__sum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. nonempty\ A. 27a \Rightarrow \forall A. 27b. nonempty\ A. 27b \Rightarrow c_2Esum_2EABS_sum\ A. 27a\ A. 27b \in ((ty_2Esum_2Esum\ A. 27a\ A. 27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 11 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 12 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a))$

Definition 13 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (7)$$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Esptree_2Elookup A_27a \in (((ty_2Eoption_2Eoption A_27a)^{(ty_2Esptree_2Espt A_27a)})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Let $c_2Esptree_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Esptree_2Edomain A_27a \in ((2^{ty_2Enum_2Enum})^{(ty_2Esptree_2Espt A_27a)}) \quad (9)$$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x))$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE\ A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt\ A_27a). (\forall V1k \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V1k)\ (ap\ (c_2Esptree_2Edomain\ A_27a)\ V0t))) \Leftrightarrow \\ (\exists V2v \in A_27a. ((ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V1k)\ V0t) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2v)))))) \end{aligned} \quad (23)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0k \in ty_2Enum_2Enum. (\\ \forall V1t \in (ty_2Esptree_2Espt\ A_27a). (((ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V0k)\ V1t) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0k)\ (ap\ (c_2Esptree_2Edomain\ A_27a)\ V1t))))))) \end{aligned}$$