

thm_2Esptree_2Elookup__inter__alt
(TMNH7U3q5jDu3zTZmWbJJJQNWw5CiJt97ci)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_COND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 11 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 12 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2Eoption_CASE$
Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (6)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b} A_27a)}) \quad (8)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2ESND$
Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (10)$$

Definition 15 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0p \in (ty_2Epair_2Epair_CASE$
Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \quad (11)$$

Let $c_2Esptree_2Einter : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esptree_2Einter A_27a A_27b \in (((ty_2Esptree_2Espt A_27a)^{(ty_2Esptree_2Espt A_27b)})^{(ty_2Esptree_2Espt A_27a)}) \quad (12)$$

Definition 16 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Definition 17 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_2E$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (13)$$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Elookup\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)} ty_2Enum_2Enum) \quad (14)$$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Esptree_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Edomain\ A_27a \in ((2^{ty_2Enum_2Enum})^{(ty_2Esptree_2Espt\ A_27a)}) \quad (15)$$

Definition 19 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\
& V5y_27))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in \\
& A_27a.(V1x = V0a)))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\
& (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V2t1) \ V3t2) = V3t2))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a).((V0opt = (c_2Eoption_2ENONE \ A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap \ (c_2Eoption_2ESOME \ A_27a) \ V1x))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap \ (ap \ (ap \ (c_2Eoption_2Eoption_CASE \\
& A_27a \ A_27b) \ (c_2Eoption_2ENONE \ A_27a)) \ V0v) \ V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap \ (ap \\
& (ap \ (c_2Eoption_2Eoption_CASE \ A_27a \ A_27b) \ (ap \ (c_2Eoption_2ESOME \\
& A_27a) \ V2x)) \ V3v) \ V4f) = (ap \ V4f \ V2x))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. (((ap\ (c.2Eoption_2ESOME\ A.27a)\ V0x) = (ap\ (c.2Eoption_2ESOME \\ & A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg((c.2Eoption_2ENONE \\ & A.27a) = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V0x)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair_2Epair_CASE\ A.27a\ A.27b \\ & A.27c)\ (ap\ (ap\ (c.2Epair_2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0m1 \in (ty_2Esptree_2Espt\ A.27a). (\forall V1m2 \in (ty_2Esptree_2Espt \\ & A.27b). (\forall V2k \in ty_2Enum_2Enum. ((ap\ (ap\ (c.2Esptree_2Elookup \\ & A.27a)\ V2k)\ (ap\ (ap\ (c.2Esptree_2Einter\ A.27a\ A.27b)\ V0m1)\ V1m2)) = \\ & (ap\ (ap\ (c.2Epair_2Epair_CASE\ (ty_2Eoption_2Eoption\ A.27a) \\ & (ty_2Eoption_2Eoption\ A.27a)\ (ty_2Eoption_2Eoption\ A.27b)) \\ & (ap\ (ap\ (c.2Epair_2E.2C\ (ty_2Eoption_2Eoption\ A.27a)\ (ty_2Eoption_2Eoption \\ & A.27b))\ (ap\ (ap\ (c.2Esptree_2Elookup\ A.27a)\ V2k)\ V0m1))\ (ap\ (ap \\ & (c.2Esptree_2Elookup\ A.27b)\ V2k)\ V1m2))) (\lambda V3v3 \in (ty_2Eoption_2Eoption \\ & A.27a). (\lambda V4v4 \in (ty_2Eoption_2Eoption\ A.27b). (ap\ (ap\ (ap\ (\\ & c.2Eoption_2Eoption_CASE\ A.27a\ (ty_2Eoption_2Eoption\ A.27a)) \\ & V3v3)\ (c.2Eoption_2ENONE\ A.27a)) (\lambda V5v \in A.27a. (ap\ (ap\ (ap\ (\\ & c.2Eoption_2Eoption_CASE\ A.27b\ (ty_2Eoption_2Eoption\ A.27a)) \\ & V4v4)\ (c.2Eoption_2ENONE\ A.27a)) (\lambda V6w \in A.27b. (ap\ (c.2Eoption_2ESOME \\ & A.27a)\ V5v)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt \\ & A.27a). (\forall V1k \in ty_2Enum_2Enum. ((p\ (ap\ (ap\ (c.2Ebool_2EIN \\ & ty_2Enum_2Enum)\ V1k)\ (ap\ (c.2Esptree_2Edomain\ A.27a)\ V0t))) \Leftrightarrow \\ & (\exists V2v \in A.27a. ((ap\ (ap\ (c.2Esptree_2Elookup\ A.27a)\ V1k) \\ & V0t) = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V2v)))))) \end{aligned} \quad (33)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in ty_2Enum_2Enum. (\forall V1t1 \in (ty_2Esptree_2Espt \\ & A_27a). (\forall V2t2 \in (ty_2Esptree_2Espt\ A_27b). ((ap\ (ap\ (c_2Esptree_2Elookup \\ & A_27a)\ V0x)\ (ap\ (ap\ (c_2Esptree_2Einter\ A_27a\ A_27b)\ V1t1)\ V2t2)) = \\ & \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ (ap \\ & (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0x)\ (ap\ (c_2Esptree_2Edomain \\ & A_27b)\ V2t2)))\ (ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V0x)\ V1t1)) \\ & \quad (c_2Eoption_2ENONE\ A_27a)))))) \end{aligned}$$