thm_2Esptree_2Elrnext__thm (TMcNyhyxTVusvRSxDkCNhgmXPWgkE8748ED)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 9 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^{A}.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Definition 16 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* $\mathsf{c\_2Ebool\_2ECOND}$ *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* $\mathsf{c\_2Eprim\_\_rec\_2EPRE}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 19 *We define* $\mathsf{c\_2Enumeral\_2EiiSUC}$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 20 *We define* $\mathsf{c\_2Enumeral\_2EiZ}$ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 21 *We define* $\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 22 *We define* $\mathsf{c\_2Erelation\_2Einv\_\_image}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27b})^{A\_27b}).\lambda V$

Definition 23 *We define* $\mathsf{c\_2Eprim\_\_rec\_2Emeasure}$ *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Erelation\_2Einv\_\_image}\ A\_27a\ t$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{10}$$

Definition 24 *We define* $\mathsf{c\_2Erelation\_2ERESTRICT}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Definition 25 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 26 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 27 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 28 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 29 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 30 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 31 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

Definition 32 *We define* c_2Erelation_2EWF *to be* $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ ($c_2Ebool_2E_21

Definition 33 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 34 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 35 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 36 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 37 *We define* c_2Esptree_2Elrnext *to be* $(ap\ (ap\ ($c_2Erelation_2EWFREC $ty\_2Enum\_2Enum\ ty\_$

Assume the following.

$$((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT2}\ \text{c\_2Earithmetic\_2EZERO})) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ \text{c\_2Earithmetic\_2EZERO})))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{15}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{16}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{18}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{19}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))) \tag{20}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{21}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V0m)))) \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V2p))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2p)))))) \tag{23}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V2p))))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow((V0m = \mathsf{c\_2Enum\_2E0})\wedge(V1n = \mathsf{c\_2Enum\_2E0}))))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{26}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V0n))) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V2p)) \Leftrightarrow ((V0m = \mathsf{c\_2Enum\_2E0}) \lor (V1n = V2p)))))) \tag{28}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1c))\ V1c) = V0a))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \land (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{31}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n)) \lor (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))) \tag{32}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{33}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2p))\ V0m))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2p))\ V1n)))))))\tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V2p))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2p)))))))\tag{35}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1r \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1r)\ V0n))\Rightarrow(\forall V2q \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2q)\ V0n))\ V1r))\ V0n) = V2q)))))\tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1q \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1q)\ V0n))\ V0n) = V1q))))\tag{37}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\forall V2z \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2z))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V1y)\ V2z))\ V0x))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1y)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0x)\ V2z)))))))))\tag{38}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Enum\_2Enum.(\forall V2b \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1a)\ V2b)))\Leftrightarrow(\forall V3d \in ty\_2Enum\_2Enum.(((V2b = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1a)\ V3d))\Rightarrow(p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})))\wedge((V1a = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2b)\ V3d))\Rightarrow(p\ (ap\ V0P\ V3d)))))))))\tag{39}$$

Assume the following.

$$True\tag{40}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{42}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{46}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\Rightarrow(p\ V1B))\Leftrightarrow((\neg(p\ V0A))\vee(p\ V1B))))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{55}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)))\wedge(\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{58}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v))\Leftrightarrow True)) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{60}$$

Assume the following.

$$(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) \wedge ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1n)) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \tag{61}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$$
$$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge$$
$$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$$
$$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge$$
$$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$$
$$\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$$
$$(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge$$
$$((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$$
$$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$$
$$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}$$
$$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$
\begin{gathered}
(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (\\
ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\\
ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\\
ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap \\
\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC} \\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (\\
ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m))))))))))))))))))))))))
\end{gathered}
\tag{63}
$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge (((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO}) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow False) \wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)) \Leftrightarrow (V0n = V1m)))))))))))) \tag{64}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))) \tag{65}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))))))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Enum\_2Enum^{A\_27a}). (p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ (ap\ (\mathsf{c\_2Eprim\_\_rec\_2Emeasure}\ A\_27a)\ V0m)))) \quad (67)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). ((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}). ((\forall V2x \in A\_27a.((\forall V3y \in A\_27a.((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A\_27a.(p\ (ap\ V1P\ V4x))))))) \quad (68)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}).(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V0f)\ V1R)\ V3z)\ V2y) = (ap\ V0f\ V2y))))))) \quad (69)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V1R)) \Rightarrow (\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{79}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow((\forall V0a \in A\_27a.((ap\ \mathsf{c\_2Esptree\_2Elrnext}\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge((\forall V1n \in ty\_2Enum\_2Enum.(\forall V2a \in A\_27b.((ap\ \mathsf{c\_2Esptree\_2Elrnext}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)) = (ap\ \mathsf{c\_2Esptree\_2Elrnext}\ V1n))))\wedge(((ap\ \mathsf{c\_2Esptree\_2Elrnext}\ \mathsf{c\_2Earithmetic\_2EZERO}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V3n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Esptree\_2Elrnext}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ \mathsf{c\_2Esptree\_2Elrnext}\ V3n))))\wedge(\forall V4n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Esptree\_2Elrnext}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V4n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ \mathsf{c\_2Esptree\_2Elrnext}\ V4n))))))))$$