

thm_2Esptree_2Emapi__Alist
(TMKKofc8XLPV4kK68s9t2J3meBwtH8MBGWZ)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (2)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (3)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (4)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone))\ (\lambda V0x \in ty_2Eone_2Eone)$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b))\ V0e$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (9)$$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a))\ (V0P)$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b))\ V0e$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a))\ V0x$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (11)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b} A_27a)}) \end{aligned} \quad (13)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP \\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (14)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (15)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (16)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (17)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esptree_2Espt\ A0) \quad (18)$$

Let $c_2Esptree_2Efoldi : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esptree_2Efoldi \\ A_27a\ A_27b \in (((A_27a^{(ty_2Esptree_2Espt\ A_27b)})^{A_27a})^{ty_2Enum_2Enum})^{((A_27a^{A_27a})^{A_27b})^{ty_2Enum_2Enum}} \end{aligned} \quad (19)$$

Definition 19 We define $c_2Esptree_2EtoAList$ to be $\lambda A_27a : \iota. (ap\ (ap\ (ap\ (c_2Esptree_2Efoldi\ (ty_2Esptree_2Espt\ A_27a)\ A_27a)\ A_27a)\ A_27a)\ A_27a)$

Let $c_2Ealist_2EALOOKUP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Ealist_2EALOOKUP \\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{A_27b})^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a))}) \end{aligned} \quad (20)$$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Elookup\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{ty_2Enum_2Enum}) \quad (21)$$

Let $c_2Esptree_2Einsert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Einsert\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{A_27a})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 20 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 21 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 22 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Definition 23 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Epair_CASE\ A_27a\ A_27b\ A_27c)$

Let $c_2Esptree_2ELN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2ELN\ A_27a \in (ty_2Esptree_2Espt\ A_27a) \quad (23)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \quad (24)$$

Definition 24 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2Ebool_CASE\ A_27a\ V0R))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (25)$$

Definition 25 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1M \in (A_27a^{A_27a}).(ap\ (c_2Erelation_2EWF\ A_27a\ V0f))$

Definition 26 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.(ap\ (c_2Erelation_2ERESTRICT\ A_27a\ V0R))$

Definition 27 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}).(ap\ (c_2Erelation_2ETC\ A_27a\ V0R))$

Definition 28 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}).(ap\ (c_2Erelation_2Eapprox\ A_27a\ V0R))$

Definition 29 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27a}).(ap\ (c_2Erelation_2Ethe_fun\ A_27a\ V0R))$

Definition 30 We define $c_2Esptree_2EfromAList$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Erelation_2EWFREC\ (ty_2Elist_2Elist\ A_27a)\ V0R))\ A_27a)$

Let $c_2Esptree_2Emapi0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esptree_2Emapi0 \\ & A_27a\ A_27b \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27b)})^{ty_2Enum_2Enum})^{((A_27a^{A_27b})^{ty_2Enum_2Enum})} \end{aligned} \quad (26)$$

Definition 31 We define $c_2Esptree_2Emapi$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum})$

Let $c_2Esptree_2Ewf : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Ewf\ A_27a \in (2^{(ty_2Esptree_2Espt\ A_27a)}) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0q \in A_27b.((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27a\ A_27b) \\ & (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ A_27b\ A_27a)))\ V0q) = (c_2Eoption_2ENONE \\ & A_27a))) \wedge (\forall V1y \in A_27a.(\forall V2x \in A_27b.(\forall V3t \in \\ & (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27b\ A_27a)).(\forall V4q \in \\ & A_27b.((ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & (ty_2Epair_2Eprod\ A_27b\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b \\ & A_27a)\ V2x)\ V1y))\ V3t))\ V4q) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ & A_27a))\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ V2x)\ V4q))\ (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y))\ (ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27a\ A_27b)\ V3t)\ V4q)))))) \end{aligned} \quad (28)$$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1)) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \\
& \hspace{10em} (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l)))))) \\
& \hspace{10em} (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{10em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME \\
& A_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE \\
& A_27b)))))) \\
& \hspace{10em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}).(\forall V1x \in (ty_2Eoption_2Eoption \\
& A_27a).(\forall V2y \in A_27b.(((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2y)) \Leftrightarrow (\exists V3z \in \\
& A_27a.((V1x = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V3z)) \wedge (V2y = (ap\ V0f \\
& V3z)))))))))) \\
& \hspace{10em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a \\
& A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \\
& \hspace{10em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((\forall V1p \in \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ & \quad A_27a. (\forall V3p_2 \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ A_27b)\ V2p_1)\ V3p_2)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0k \in ty_2Enum_2Enum. (\forall V1f \in ((A_27a^{A_27b})^{ty_2Enum_2Enum}). \\ & \quad (\forall V2pt \in (ty_2Esptree_2Espt\ A_27b). ((ap\ (ap\ (c_2Esptree_2Elookup \\ & \quad A_27a)\ V0k)\ (ap\ (ap\ (c_2Esptree_2Emapi\ A_27a\ A_27b)\ V1f)\ V2pt)) = \\ & \quad (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27b\ A_27a)\ (ap\ V1f\ V0k))\ (ap \\ & \quad (ap\ (c_2Esptree_2Elookup\ A_27b)\ V0k)\ V2pt)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt \\ & \quad A_27a). (\forall V1x \in ty_2Enum_2Enum. ((ap\ (ap\ (c_2Ealist_2EALOOKUP \\ & \quad A_27a\ ty_2Enum_2Enum)\ (ap\ (c_2Esptree_2EtoAList\ A_27a)\ V0t)) \\ & \quad V1x) = (ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V1x)\ V0t)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in (ty_2Esptree_2Espt \\ & \quad A_27a). (\forall V1t2 \in (ty_2Esptree_2Espt\ A_27a). (((p\ (ap\ (c_2Esptree_2Ewf \\ & \quad A_27a)\ V0t1)) \wedge (p\ (ap\ (c_2Esptree_2Ewf\ A_27a)\ V1t2))) \Rightarrow ((V0t1 = \\ & \quad V1t2) \Leftrightarrow (\forall V2n \in ty_2Enum_2Enum. ((ap\ (ap\ (c_2Esptree_2Elookup \\ & \quad A_27a)\ V2n)\ V0t1) = (ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V2n)\ V1t2)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ls \in (ty_2Elist_2Elist \\ & \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A_27a). (\forall V1x \in ty_2Enum_2Enum. \\ & \quad ((ap\ (ap\ (c_2Esptree_2Elookup\ A_27a)\ V1x)\ (ap\ (c_2Esptree_2EfromAList \\ & \quad A_27a)\ V0ls)) = (ap\ (ap\ (c_2Ealist_2EALOOKUP\ A_27a\ ty_2Enum_2Enum) \\ & \quad V0ls)\ V1x)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ls \in (ty_2Elist_2Elist \\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A.27a)).(p\ (ap\ (c_2Esptree_2Ewf \\ A.27a)\ (ap\ (c_2Esptree_2EfromAList\ A.27a)\ V0ls)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in ((A.27a^{A.27b})^{ty_2Enum_2Enum}).(\forall V1pt \in (ty_2Esptree_2Espt \\ A.27b).(p\ (ap\ (c_2Esptree_2Ewf\ A.27a)\ (ap\ (ap\ (c_2Esptree_2Emapi \\ A.27a\ A.27b)\ V0f)\ V1pt)))))) \end{aligned} \quad (55)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in ((A.27a^{A.27b})^{ty_2Enum_2Enum}).(\forall V1pt \in (ty_2Esptree_2Espt \\ A.27b).((ap\ (ap\ (c_2Esptree_2Emapi\ A.27a\ A.27b)\ V0f)\ V1pt) = (ap \\ (c_2Esptree_2EfromAList\ A.27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod \\ ty_2Enum_2Enum\ A.27b)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A.27a)) \\ (\lambda V2kv \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A.27b).(ap\ (ap\ (\\ c_2Epair_2E2C\ ty_2Enum_2Enum\ A.27a)\ (ap\ (c_2Epair_2EFST\ ty_2Enum_2Enum \\ A.27b)\ V2kv))\ (ap\ (ap\ V0f\ (ap\ (c_2Epair_2EFST\ ty_2Enum_2Enum\ A.27b) \\ V2kv))\ (ap\ (c_2Epair_2ESND\ ty_2Enum_2Enum\ A.27b)\ V2kv))))))\ (ap \\ (c_2Esptree_2EtoAList\ A.27b)\ V1pt)))))) \end{aligned}$$