

# thm\_2Esptree\_2Espt\_\_acc\_\_0 (TM- cetG6QuMoRwzVdnKthKPGoN4hybGF6DqE)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda \tau a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)) (\lambda V3x \in 2.V3x))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2ESUC\_REP)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 6** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2ESUC))$

**Definition 7** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Enum\_E0$ .

**Definition 8** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_E$

**Definition 9** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 10** We define  $c\_Emin\_E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_Ebool\_E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E\_21 2) (\lambda V2t \in$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_Earithmetic\_E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 12** We define  $c\_Ebool\_E\_EF$  to be  $(ap (c\_Ebool\_E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_Emin\_E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_Ebool\_E\_COND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 15** We define  $c\_Ecombin\_EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 16** We define  $c\_Ecombin\_ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 17** We define  $c\_Ecombin\_EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_Ecombin\_ES A\_27a (A\_27a^{A\_27a}) A$

**Definition 18** We define  $c\_Ebool\_E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E\_3D\_3D\_3E V0t) c\_Ebool\_E$

**Definition 19** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_E\_40$

**Definition 20** We define  $c\_Erelation\_EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_Ebool\_E\_21$

Let  $c\_Ebool\_EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Ebool\_EARB A\_27a \in A\_27a \quad (10)$$

**Definition 21** We define  $c\_Erelation\_ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1f \in (A\_27b^{A\_27a})$

**Definition 22** We define  $c\_Erelation\_ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in A\_27a$

**Definition 23** We define  $c\_Erelation\_Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 24** We define  $c\_Erelation\_Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 25** We define  $c\_Erelation\_EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 26** We define  $c\_Esptree\_Elrnext$  to be  $(ap (ap (c\_Erelation\_EWFREC ty\_2Enum\_2Enum ty\_2Enum\_2Enum$

Let  $c\_Esptree\_Espt\_acc : \iota$  be given. Assume the following.

$$c\_Esptree\_Espt\_acc \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_Earithmetic\_2E\_2A V0m) (ap c\_Earithmetic\_2ENUMERAL (ap c\_Earithmetic\_2EBIT1 c\_Earithmetic\_2EZERO)))) = V0m)) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_Earithmetic\_2E\_2A V1n) V0m)))) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\
& (\forall V0a \in A_{.27a}.((ap\ c_{.2}Esptree_{.2}Elrnext\ c_{.2}Enum_{.2}E0) = ( \\
ap\ c_{.2}Earithmetic_{.2}ENUMERAL\ (ap\ c_{.2}Earithmetic_{.2}EBIT1\ c_{.2}Earithmetic_{.2}EZERO)))) \wedge \\
& ((\forall V1n \in ty_{.2}Enum_{.2}Enum.(\forall V2a \in A_{.27b}.((ap\ c_{.2}Esptree_{.2}Elrnext \\
(ap\ c_{.2}Earithmetic_{.2}ENUMERAL\ V1n)) = (ap\ c_{.2}Esptree_{.2}Elrnext \\
V1n)))) \wedge (((ap\ c_{.2}Esptree_{.2}Elrnext\ c_{.2}Earithmetic_{.2}EZERO) = \\
(ap\ c_{.2}Earithmetic_{.2}ENUMERAL\ (ap\ c_{.2}Earithmetic_{.2}EBIT1\ c_{.2}Earithmetic_{.2}EZERO)))) \wedge \\
& ((\forall V3n \in ty_{.2}Enum_{.2}Enum.((ap\ c_{.2}Esptree_{.2}Elrnext\ (ap\ c_{.2}Earithmetic_{.2}EBIT1 \\
V3n)) = (ap\ (ap\ c_{.2}Earithmetic_{.2}E_{.2}A\ (ap\ c_{.2}Earithmetic_{.2}ENUMERAL \\
(ap\ c_{.2}Earithmetic_{.2}EBIT2\ c_{.2}Earithmetic_{.2}EZERO)))\ (ap\ c_{.2}Esptree_{.2}Elrnext \\
V3n)))) \wedge (\forall V4n \in ty_{.2}Enum_{.2}Enum.((ap\ c_{.2}Esptree_{.2}Elrnext \\
(ap\ c_{.2}Earithmetic_{.2}EBIT2\ V4n)) = (ap\ (ap\ c_{.2}Earithmetic_{.2}E_{.2}A \\
(ap\ c_{.2}Earithmetic_{.2}ENUMERAL\ (ap\ c_{.2}Earithmetic_{.2}EBIT2\ c_{.2}Earithmetic_{.2}EZERO))) \\
(ap\ c_{.2}Esptree_{.2}Elrnext\ V4n)))))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty_{.2}Enum_{.2}Enum.(\forall V1i \in ty_{.2}Enum_{.2}Enum.( \\
(ap\ (ap\ c_{.2}Esptree_{.2}Espt_{.2}acc\ V1i)\ V0k) = (ap\ (ap\ c_{.2}Earithmetic_{.2}E_{.2}B \\
(ap\ (ap\ c_{.2}Earithmetic_{.2}E_{.2}A\ (ap\ c_{.2}Esptree_{.2}Elrnext\ V1i))\ V0k)) \\
V1i))))))
\end{aligned} \tag{18}$$

**Theorem 1**

$$(\forall V0k \in ty_{.2}Enum_{.2}Enum.((ap\ (ap\ c_{.2}Esptree_{.2}Espt_{.2}acc\ c_{.2}Enum_{.2}E0)\ V0k) = V0k))$$