thm_2Esptree_2Espt__center__def (TMLSa1ETvAmPFvPUiNVsnSTTZiJZHcWwPfD)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Erelation_2EEMPTY__REL *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27a.\mathsf{c\_2Eb}$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{1}$$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Let $ty\_2Esptree\_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esptree\_2Espt\ A0) \tag{2}$$

Let $c\_2Esptree\_2EBS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2EBS\ A\_27a \in ((((ty\_2Esptree\_2Espt\ A\_27a)^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)}) \tag{3}$$

Let $c\_2Esptree\_2EBN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2EBN\ A\_27a \in (((ty\_2Esptree\_2Espt\ A\_27a)^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_27a)}) \quad (4)$$

Let $c\_2Esptree\_2ELS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2ELS\ A\_27a \in ((ty\_2Esptree\_2Espt\ A\_27a)^{A\_27a}) \quad (5)$$

Let $c\_2Esptree\_2ELN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2ELN\ A\_27a \in (ty\_2Esptree\_2Espt\ A\_27a) \quad (6)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (7)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (8)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (9)$$

Definition 11 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (10)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (11)$$

Definition 12 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 13 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 15 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 16 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 17 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 18 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 19 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Esptree\_2Espt\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2E$$
$$A\_27a\ A\_27b \in (((((A\_27b^{(((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)})})^{((A\_27b^{(ty\_2Esptree\_2Espt\ A\_}}$$

(12)

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 21 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Definition 22 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 23 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 24 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 25 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 26 *We define* c_2Esptree_2Espt__center *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ (ty\_2E$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V1R)) \Rightarrow (\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2f1 \in ((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_27a)}). (\forall V3f2 \in (((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)}). ((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Esptree\_2Espt\_\_CASE\ A\_27a\ A\_27b)\ (c\_2Esptree\_2ELN\ A\_27a))\ V0v)\ V1f)\ V2f1)\ V3f2) = V0v)))))\wedge((\forall V4a \in A\_27a.(\forall V5v \in A\_27b.(\forall V6f \in (A\_27b^{A\_27a}).(\forall V7f1 \in ((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_27a)}). (\forall V8f2 \in (((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)}). ((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Esptree\_2Espt\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (c\_2Esptree\_2ELS\ A\_27a)\ V4a))\ V5v)\ V6f)\ V7f1)\ V8f2) = (ap\ V6f\ V4a)))))))\wedge((\forall V9a0 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V10a1 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V11v \in A\_27b.(\forall V12f \in (A\_27b^{A\_27a}).(\forall V13f1 \in ((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_27a)}). (\forall V14f2 \in (((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)}). ((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Esptree\_2Espt\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Esptree\_2EBN\ A\_27a)\ V9a0)\ V10a1))\ V11v)\ V12f)\ V13f1)\ V14f2) = (ap\ (ap\ V13f1\ V9a0)\ V10a1))))))))\wedge(\forall V15a0 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V16a1 \in A\_27a.(\forall V17a2 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V18v \in A\_27b.(\forall V19f \in (A\_27b^{A\_27a}).(\forall V20f1 \in ((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_27a)}). (\forall V21f2 \in (((A\_27b^{(ty\_2Esptree\_2Espt\ A\_27a)})^{A\_27a})^{(ty\_2Esptree\_2Espt\ A\_27a)}). ((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Esptree\_2Espt\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Esptree\_2EBS\ A\_27a)\ V15a0)\ V16a1)\ V17a2))\ V18v)\ V19f)\ V20f1)\ V21f2) = (ap\ (ap\ (ap\ V21f2\ V15a0)\ V16a1)\ V17a2)))))))))))) \quad (16)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1t1 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V2t2 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V3v1 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V4v2 \in (ty\_2Esptree\_2Espt\ A\_27a).(((ap\ (\mathsf{c\_2Esptree\_2Espt\_\_center}\ A\_27a)\ (ap\ (c\_2Esptree\_2ELS\ A\_27a)\ V0x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x))\wedge(((ap\ (\mathsf{c\_2Esptree\_2Espt\_\_center}\ A\_27a)\ (ap\ (ap\ (ap\ (c\_2Esptree\_2EBS\ A\_27a)\ V1t1)\ V0x)\ V2t2)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x))\wedge(((ap\ (\mathsf{c\_2Esptree\_2Espt\_\_center}\ A\_27a)\ (c\_2Esptree\_2ELN\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\wedge((ap\ (\mathsf{c\_2Esptree\_2Espt\_\_center}\ A\_27a)\ (ap\ (ap\ (c\_2Esptree\_2EBN\ A\_27a)\ V3v1)\ V4v2)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))))))))))$$