thm_2Esptree_2Esubspt__domain (TMUanzoGLHi1FhNh72fb6a7BTF7YLu3b9Z7)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (\mathsf{c}$

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $ty\_2Esptree\_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esptree\_2Espt\ A0) \tag{3}$$

Let $c\_2Esptree\_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2Edomain\ A\_27a \in ((2^{ty\_2Enum\_2Enum})^{(ty\_2Esptree\_2Espt\ A\_27a)}) \quad (4)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (5)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (6)$$

Definition 11 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (7)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (8)$$

Definition 12 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Esptree\_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2Elookup\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esptree\_2Espt\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $c\_2Esptree\_2Esubspt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2Esubspt\ A\_27a \in ((2^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_2} \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{16}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{17}$$

Assume the following.

$$(\forall V0v \in ty\_2Eone\_2Eone.(V0v = \mathsf{c\_2Eone\_2Eone})) \tag{18}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Eone\_2Eone}).((\forall V1x \in ty\_2Eone\_2Eone.(p\ (ap\ V0P\ V1x))) \Leftrightarrow (p\ (ap\ V0P\ \mathsf{c\_2Eone\_2Eone})))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ (c\_2Esptree\_2Edomain\ A\_27a)\ V0t))) \Leftrightarrow (\exists V2v \in A\_27a.((ap\ (ap\ (c\_2Esptree\_2Elookup\ A\_27a)\ V1k)\ V0t) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2v)))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V1t2 \in (ty\_2Esptree\_2Espt\ A\_27a).((p\ (ap\ (ap\ (c\_2Esptree\_2Esubspt\ A\_27a)\ V0t1)\ V1t2)) \Leftrightarrow (\forall V2x \in ty\_2Enum\_2Enum.(\forall V3y \in A\_27a.(((ap\ (ap\ (c\_2Esptree\_2Elookup\ A\_27a)\ V2x)\ V0t1) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3y)) \Rightarrow ((ap\ (ap\ (c\_2Esptree\_2Elookup\ A\_27a)\ V2x)\ V1t2) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3y)))))))) \tag{21}$$

Theorem 1

$$(\forall V0t1 \in (ty\_2Esptree\_2Espt\ ty\_2Eone\_2Eone).(\forall V1t2 \in (ty\_2Esptree\_2Espt\ ty\_2Eone\_2Eone).((p\ (ap\ (ap\ (c\_2Esptree\_2Esubspt\ ty\_2Eone\_2Eone)\ V0t1)\ V1t2)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Esptree\_2Edomain\ ty\_2Eone\_2Eone)\ V0t1))\ (ap\ (c\_2Esptree\_2Edomain\ ty\_2Eone\_2Eone)\ V1t2))))))$$