thm_2Esptree_2Esubspt__refl (TMP6x8GcKxM5prvZ884jQoaG45taqUS6KDa)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{1}$$

Let $ty\_2Esptree\_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esptree\_2Espt\ A0) \tag{2}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{3}$$

Let $c\_2Esptree\_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2Elookup\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esptree\_2Espt\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{4}$$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Esptree\_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esptree\_2Edomain\ A\_27a \in ((2^{ty\_2Enum\_2Enum})^{(ty\_2Esptree\_2Espt\ A\_27a)}) \tag{5}$$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x)))$

Let $c\_2Esptree\_2Esubspt:\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Esptree\_2Esubspt\ A\_27a\in((2^{(ty\_2Esptree\_2Espt\ A\_27a)})^{(ty\_2Esptree\_2Espt\ A\_2} \tag{6}$$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t\in 2.((\forall V1x\in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{8}$$

Assume the following.

$$(\forall V0t\in 2.(((True\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\wedge True){\Leftrightarrow}(p\ V0t))\wedge(((False\wedge(p\ V0t)){\Leftrightarrow}False)\wedge((((p\ V0t)\wedge False){\Leftrightarrow}False)\wedge(((p\ V0t)\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{9}$$

Assume the following.

$$(\forall V0t\in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x\in A\_27a.((V0x=V0x){\Leftrightarrow}True)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x\in A\_27a.(\forall V1y\in A\_27a.((V0x=V1y){\Leftrightarrow}(V1y=V0x)))) \tag{12}$$

Assume the following.

$$(\forall V0t\in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}(p\ V0t))\wedge(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t)))\wedge(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))) \tag{13}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(\forall V2t3\in 2.(((p\ V0t1){\Rightarrow}((p\ V1t2){\Rightarrow}(p\ V2t3))){\Leftrightarrow}(((p\ V0t1)\wedge(p\ V1t2)){\Rightarrow}(p\ V2t3)))))) \tag{14}$$

Assume the following.

$$(\forall V0x\in 2.(\forall V1x\_27\in 2.(\forall V2y\in 2.(\forall V3y\_27\in 2.((((p\ V0x){\Leftrightarrow}(p\ V1x\_27))\wedge((p\ V1x\_27){\Rightarrow}((p\ V2y){\Leftrightarrow}(p\ V3y\_27)))){\Rightarrow}(((p\ V0x){\Rightarrow}(p\ V2y)){\Leftrightarrow}((p\ V1x\_27){\Rightarrow}(p\ V3y\_27)))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp1 \in (ty\_2Esptree\_2Espt\ A\_27a).(\forall V1sp2 \in (ty\_2Esptree\_2Espt\ A\_27a).((p\ (ap\ (ap\ (c\_2Esptree\_2Esubspt\ A\_27a)\ V0sp1)\ V1sp2)) \Leftrightarrow (\forall V2k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V2k)\ (ap\ (c\_2Esptree\_2Edomain\ A\_27a)\ V0sp1))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V2k)\ (ap\ (c\_2Esptree\_2Edomain\ A\_27a)\ V1sp2))) \land ((ap\ (ap\ (c\_2Esptree\_2Elookup\ A\_27a)\ V2k)\ V1sp2) = (ap\ (ap\ (c\_2Esptree\_2Elookup\ A\_27a)\ V2k)\ V0sp1)))))))) \tag{16}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0sp \in (ty\_2Esptree\_2Espt\ A\_27a).(p\ (ap\ (ap\ (c\_2Esptree\_2Esubspt\ A\_27a)\ V0sp)\ V0sp)))$$