



Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (4)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

**Definition 11** We define  $c\_2Estate\_transformer\_2EUNIT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27b.(\lambda V1s \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1s))$

**Definition 12** We define  $c\_2Estate\_transformer\_2EBIND$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0g \in ((ty\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0g)$

**Definition 13** We define  $c\_2Estate\_transformer\_2EMMAP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27c)$

**Definition 14** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 15** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c)^{A\_27b})^{A\_27a})$

**Definition 16** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a)^{A\_27a})\ V0f))$

**Definition 17** We define  $c\_2Estate\_transformer\_2EJOIN$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0z \in ((ty\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0z)$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27b)^{A\_27a}. \\ (\forall V1g \in (A\_27a)^{A\_27c}.(\forall V2h \in (A\_27c)^{A\_27d}).((ap\ ( \\ ap\ (c\_2Ecombin\_2Eo\ A\_27d\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (c\_2Ecombin\_2Eo \\ A\_27d\ A\_27a\ A\_27c)\ V1g)\ V2h)) = (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27d\ A\_27b \\ A\_27c)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g))\ V2h)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b)^{A\_27a}.(((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\ A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0g \in (( \\
& \quad (ty\_2Epair\_2Eprod\ A.27c\ A.27a)^{A.27a})^{A.27d}). (\forall V1f \in (( \\
& \quad (ty\_2Epair\_2Eprod\ A.27d\ A.27a)^{A.27a})^{A.27b}). ((ap\ (c.2Estate\_transformer\_2EEXT \\
& \quad A.27b\ A.27c\ A.27a)\ (ap\ (ap\ (c.2Estate\_transformer\_2EMCOMP\ A.27b \\
& \quad A.27d\ A.27c\ A.27a)\ V0g)\ V1f)) = (ap\ (ap\ (c.2Ecombin\_2Eo\ ((ty\_2Epair\_2Eprod \\
& \quad A.27b\ A.27a)^{A.27a})\ ((ty\_2Epair\_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty\_2Epair\_2Eprod \\
& \quad A.27d\ A.27a)^{A.27a}))\ (ap\ (c.2Estate\_transformer\_2EEXT\ A.27d \\
& \quad A.27c\ A.27a)\ V0g))\ (ap\ (c.2Estate\_transformer\_2EEXT\ A.27b\ A.27d \\
& \quad A.27a)\ V1f))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in (((ty\_2Epair\_2Eprod\ A.27c\ A.27b)^{A.27b})^{A.27a}). \\
& \quad ((ap\ (ap\ (c.2Ecombin\_2Eo\ A.27a\ ((ty\_2Epair\_2Eprod\ A.27c\ A.27b)^{A.27b}) \\
& \quad (ty\_2Epair\_2Eprod\ A.27a\ A.27b)^{A.27b}))\ (ap\ (c.2Estate\_transformer\_2EEXT \\
& \quad A.27a\ A.27c\ A.27b)\ V0f))\ (c.2Estate\_transformer\_2EUNIT\ A.27b \\
& \quad A.27a)) = V0f))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}). ((ap\ (c.2Estate\_transformer\_2EMMAP \\
& \quad A.27a\ A.27c\ A.27b)\ V0f) = (ap\ (c.2Estate\_transformer\_2EEXT\ A.27b \\
& \quad A.27c\ A.27a)\ (ap\ (ap\ (c.2Ecombin\_2Eo\ A.27b\ ((ty\_2Epair\_2Eprod \\
& \quad A.27c\ A.27a)^{A.27a})\ A.27c)\ (c.2Estate\_transformer\_2EUNIT\ A.27a \\
& \quad A.27c))\ V0f))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad (c.2Estate\_transformer\_2EJOIN\ A.27a\ A.27b) = (ap\ (c.2Estate\_transformer\_2EEXT \\
& \quad ((ty\_2Epair\_2Eprod\ A.27b\ A.27a)^{A.27a})\ A.27b\ A.27a)\ (c.2Ecombin\_2EI \\
& \quad ((ty\_2Epair\_2Eprod\ A.27b\ A.27a)^{A.27a}))))))
\end{aligned} \tag{13}$$

### Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in (((ty\_2Epair\_2Eprod\ A.27c\ A.27a)^{A.27a})^{A.27b}). \\
& \quad ((ap\ (c.2Estate\_transformer\_2EEXT\ A.27b\ A.27c\ A.27a)\ V0f) = ( \\
& \quad ap\ (ap\ (c.2Ecombin\_2Eo\ ((ty\_2Epair\_2Eprod\ A.27b\ A.27a)^{A.27a}) \\
& \quad ((ty\_2Epair\_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty\_2Epair\_2Eprod\ (( \\
& \quad ty\_2Epair\_2Eprod\ A.27c\ A.27a)^{A.27a})\ A.27a)^{A.27a}))\ (c.2Estate\_transformer\_2EJOIN \\
& \quad A.27a\ A.27c))\ (ap\ (c.2Estate\_transformer\_2EMMAP\ A.27a\ ((ty\_2Epair\_2Eprod \\
& \quad A.27c\ A.27a)^{A.27a})\ A.27b)\ V0f))))))
\end{aligned}$$