

thm_2Estate__transformer_2EEXT__MCOMP
(TM-
JEUgkMo7CD6UpoYkNA1iRWZSMD8u6Z4sS)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 6 We define $c_2Estate_transformer_2EEXT$ to be $\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s : \iota.\lambda V0f \in (((ty_$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b} A_27a)}) \end{aligned} \quad (4)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ V0x\ V1y))$

Definition 10 We define $c_2Epair_2ECURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27c^{(ty_2Epair_2EABS_prod\ A_27a\ A_27b)})$

Definition 11 We define $c_2Estate_transformer_2EMCOMP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27s :$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ (\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a) \\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). \\ ((ap\ (c_2Epair_2EUNCURRY\ A_27a\ A_27b\ A_27c)\ (ap\ (c_2Epair_2ECURRY \\ A_27a\ A_27b\ A_27c)\ V0f)) = V0f)) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0g \in ((\\
& \quad (ty_2Epair_2Eprod\ A.27c\ A.27b)^{A.27b})^{A.27d}). (\forall V1f \in ((\\
& (ty_2Epair_2Eprod\ A.27d\ A.27b)^{A.27b})^{A.27a}). ((ap\ (ap\ (c_2Estate_transformer_2EMCOMP \\
& \quad A.27a\ A.27d\ A.27c\ A.27b)\ V0g)\ V1f) = (ap\ (c_2Epair_2ECURRY\ A.27a \\
& \quad A.27b\ (ty_2Epair_2Eprod\ A.27c\ A.27b))\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad (ty_2Epair_2Eprod\ A.27a\ A.27b)\ (ty_2Epair_2Eprod\ A.27c\ A.27b) \\
& \quad (ty_2Epair_2Eprod\ A.27d\ A.27b))\ (ap\ (c_2Epair_2EUNCURRY\ A.27d \\
& \quad A.27b\ (ty_2Epair_2Eprod\ A.27c\ A.27b))\ V0g))\ (ap\ (c_2Epair_2EUNCURRY \\
& \quad A.27a\ A.27b\ (ty_2Epair_2Eprod\ A.27d\ A.27b))\ V1f)))))) \\
& \hspace{15em} (11)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0g \in ((\\
& \quad (ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})^{A.27d}). (\forall V1f \in ((\\
& (ty_2Epair_2Eprod\ A.27d\ A.27a)^{A.27a})^{A.27b}). ((ap\ (c_2Estate_transformer_2EEXT \\
& \quad A.27b\ A.27c\ A.27a)\ (ap\ (ap\ (c_2Estate_transformer_2EMCOMP\ A.27b \\
& \quad A.27d\ A.27c\ A.27a)\ V0g)\ V1f)) = (ap\ (ap\ (c_2Ecombin_2Eo\ ((ty_2Epair_2Eprod \\
& \quad A.27b\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod\ A.27c\ A.27a)^{A.27a})\ ((ty_2Epair_2Eprod \\
& \quad A.27d\ A.27a)^{A.27a}))\ (ap\ (c_2Estate_transformer_2EEXT\ A.27d \\
& \quad A.27c\ A.27a)\ V0g))\ (ap\ (c_2Estate_transformer_2EEXT\ A.27b\ A.27d \\
& \quad A.27a)\ V1f))))))
\end{aligned}$$